PRACTICAL DESIGNING IN REINFORCED CONCRETE

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A DESCRIPTION OF REINFORCED CONCRETE AND ITS SUITABILITY FOR VARIOUS KINDS OF STRUCTURES, TOGETHER WITH EXAMPLES WORKED OUT IN DETAIL FOR ALL TYPES OF BEAMS, FLOORS AND COLUMNS

BY

M. T. CANTELL

Assoc. Surveyors' Inst., Eng. (1917); Licentiate Royal Inst. British Architects (1910); Member Amer. Soc. Mechanical Engineers (1918), Assoc. Engineering Inst. Canada (1920); Member Manitoba Assoc. Professional Engineers (1920); Member Canadian Inst. Mining and Metallurgy (1921), Fellow Royal Society of Arts (1922), Fourteen years Head of Architecture and Building Dept. Municipal Technical College, Brighton, England, Certificated Architect, State of California, U S A

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PREFACE

Since the publication of his previous work, "Reinforced Concrete Construction," in two volumes, the author has made an extensive study of this class of work in the United States and Canada, and has been responsible for the design and construction of a number of civil engineering and architectural structures in those countries, consequently the present work embodies the results of this experience as well as that previously gained in England. The work is written to comply as far as possible with American, Canadian and British conditions and practice, and to agree with the recommendations of the reports of the Royal Institute of British Architects, the Engineering Institute of Canada, and the American Joint Committee on Concrete and Reinforced Concrete.

After careful consideration of the symbols used by various authorities, the author has adopted the same system as used in his previous works, this being considered the most suitable for the subject and the easiest to follow, as in most cases the symbol is the initial letter of the word it represents.

The author having many years teaching experience in technical institutions is well acquainted with the difficulties attending the use of algebraical formulas by building construction students, and others, who while having practical experience have only an elementary knowledge of mathematics and

mechanics, and has therefore explained the use of all formulas in as simple practical terms as possible with a view to making the work valuable as a college text-book, as well as a useful reference book for architects engineers and others directly associated with the design and construction of works in reinforced concrete.

The work is divided into two parts. Part I. contains a description of reinforced concrete, the purposes for which it is suited, the causes of failures and how to avoid them, the selection and properties of the component materials, the principles of design, general information and data for designing and construction, examples of designing all kinds of beams, floors and columns. Part II. contains more advanced examples of the work included in Part I., and in addition many fully worked out and illustrated examples of all types of structures, including those subject to water, earth, grain, and wind pressures.

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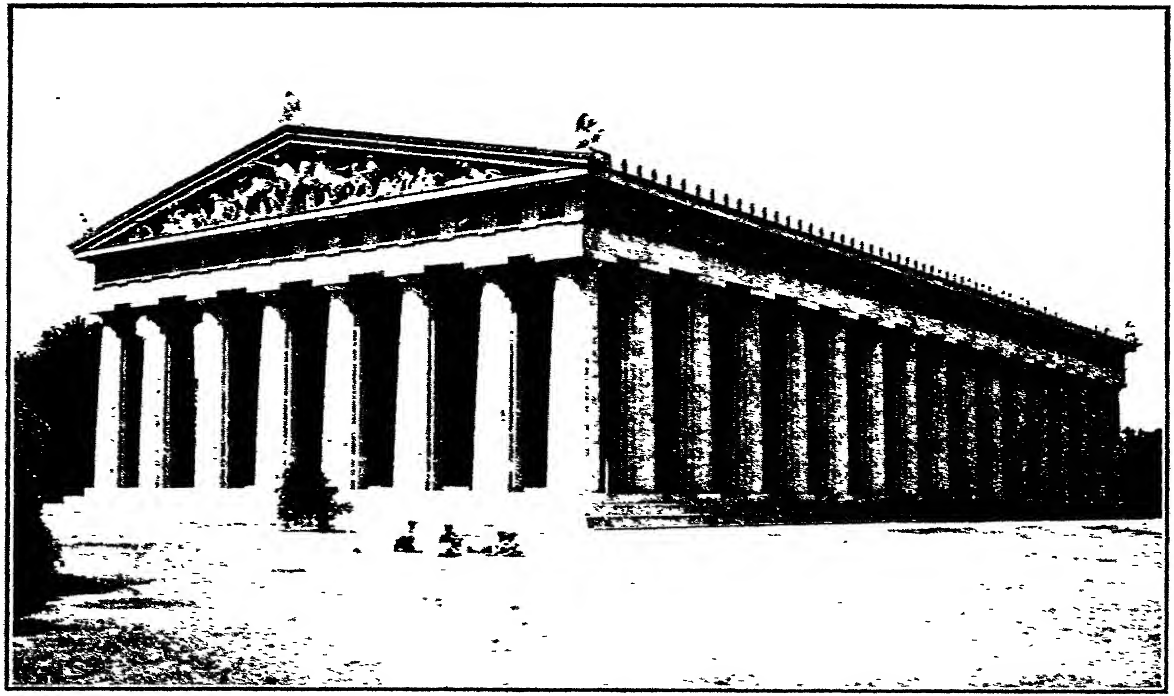
PRACTICAL DESIGNING IN REINFORCED CONCRETE

Reinforced concrete is a combination of Portland Cement, Concrete and Steel, two of the most important materials used for building construction, which, if used separately, have each their advantages and disadvantages; if combined, of the right quality and properly proportioned, the resulting material will have the advantages of both and the disadvantages of neither.

Concrete. Concrete has great strength in compression, its crushing resistance when a month old being approximately 2,500 lb. per square inch. It is extremely durable in water or air, being practically everlasting, and its strength increases with age, but it is of little value for use where tensional resistance is required, as it is unable to resist more than about 200 lb. per square inch; it is a good material to resist heat, but is not ductile, owing to the lack of this property and to its tensional weakness it cannot resist the slightest contraction, which will take place under variations of temperature, without developing cracks. Expansion and contraction is sure to take place in this as in all other materials, although to a less extent than in most others. These cracks entirely destroy the slight tensional resistance the concrete might otherwise offer; consequently, concrete alone can be used only for such structures or parts of structures that can be designed to act in compression, and this, in many



THE PARTHENON, NASHVILLE, TENNESSEE, U.S.A.
A reproduction of the Athens Parthenon.

An example showing the [illegible] of Reinforced Concrete [illegible] Classic Architecture.

Russell E. Hart, Architect.

cases, necessitates a very large mass of concrete, resulting in a much greater weight and demand on space than is desirable. As an example of this, take a retaining wall about 20 feet high. If built of plain concrete it would be about 9 feet thick at the base. This great thickness is necessary to prevent tension taking place on one side of the wall. If built of reinforced concrete the thickness at the base would be from 9 to 18 inches, according to the design adopted. Therefore, an obvious disadvantage of plain concrete is its great mass and consequent weight and demand on space. Further advantages are its adaptability to various architectural and structural forms ; its low cost of maintenance, and its resistance to electrolytic action. The latter, until recently, was considered doubtful, but according to an extensive series of very careful experiments continued over a period of several months, which were described in a paper read at a meeting of the American Institute of Electrical Engineers, the strength of concrete is not affected by electrolytic action, failures in reinforced concrete reported to be due to this action were found to be entirely due to the forces produced by increase of volume when the iron was changed into iron oxide, and not by any direct action of electric current upon the concrete. A summary of the tests was given as follows :--

(a) The compressive strength of fresh water cement cubes was not affected by an average current density of 1·2 milliamperes per square inch, applied for several days.

(b) The compressive strength of fresh water concrete cubes was not affected by an average current density of 1·8 milliamperes per square inch applied for 225 days.

(c) The compressive strength of salt water cement cubes was not affected by an average current density of 10·2 milliamperes per square inch applied for 113 days.

(d) The compressive strength of salt water concrete cubes was not affected by an average current density of 13·8 milliamperes per square inch applied for 110 days.

This, together with similar results of other authorita-

tive tests, may be considered conclusive evidence that Portland cement concrete is suitable for use in a locality subject to stray electric currents.

Steel.—In steel we have a material of great strength in both compression and tension, its ultimate resistance being as much as from 60,000 lb. to 100,000 lb. per

FIG. 1.—FURNITURE STORAGE WAREHOUSE OF REINFORCED CONCRETE HOLLYWOOD, CALIFORNIA.

Morgan, Walls and Clements, Architects, Los Angeles.

An example illustrating the suitability of this class of construction for all warehouse buildings.

square inch; but its strength diminishes with age, which is chiefly due to oxidation that takes place on exposure to moisture or to atmospheric influence. Such oxidation is very detrimental to its strength, even a twentieth of an inch of rust on a $\frac{3}{8}$ inch plate or bar will diminish its strength by 13 per cent.

Another disadvantage of steel is its excessive expansion and loss of strength under a high temperature. It gains in strength with increase of heat up to about 500 degrees F.; beyond this it rapidly diminishes in strength. With a rise of temperature from the normal up to 500 degrees F. a beam 28 feet long will expand one inch. At 1,000 degrees it will expand 2 inches or 1 inch in 14 feet. In ordinary house fires the temperature seldom exceeds 1,000 degrees, but in large buildings it is known to have exceeded 2,500 degrees. Under these conditions the resistance offered by the load and the fixed ends of beams columns and other parts of steel frame structures, prevents longitudinal expansion, and thus causes an increase of stress far beyond what the material is capable of resisting, especially in its weakened condition, the beam or column therefore buckles and causes collapse of the structure.

Further disadvantages of steel structures are the high cost of maintenance and the fragile appearance, the latter, especially with exposed frame structures, suggest weakness rather than strength.

CONCRETE AND STEEL COMBINED.

In concrete and steel combined we have the great strength, toughness, and rigidity of the steel; the great durability and fire resistance of concrete; the appearance of stability and strength; no loss of strength with age; saving of cost in construction; illimination of maintenance costs; lower insurance rates on the building, and its contents; a material adaptable to all forms of architectural and structural work, and one suitable for structures above ground, below ground or under water, for the construction of workmen's cottages to

millionaires' mansions, for business premises, hotels, churches, theatres, public buildings, factories, reservoirs, water mains and conduits, sewers, tunnels, grain elevators, roads, bridges, sea defence works, monumental works and numerous other purposes. It is a

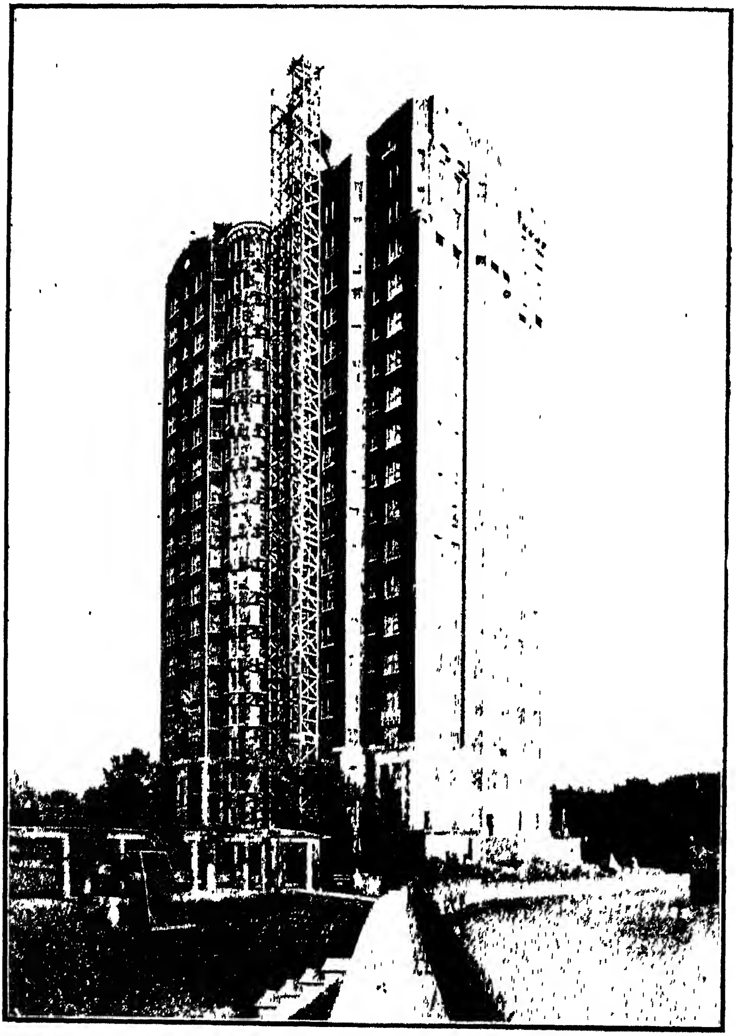


FIG. 2.—DETROIT TOWERS APARTMENTS, DETROIT, MICH., U.S.A.
A 19-storey reinforced concrete building, an example illustrating the suitability of this class of construction for tall residential buildings.

method of construction which is fast superseding that of wood, brick, stone and steel, and is far superior to any of these in resisting water, fire, earthquakes and atmospheric influence.

Earthquake Resistance.—Evidence of the superior earthquake resistance of reinforced concrete was seen in Japan after the severe earthquake on September 1, 1923. It was reported by the Tokyo Building Department that of a total of 592 reinforced concrete buildings in the city of Tokyo, 8 collapsed, 11 partially collapsed, 42 suffered severe but reparable damage, 60 suffered small damage, and 462 were not damaged at all.

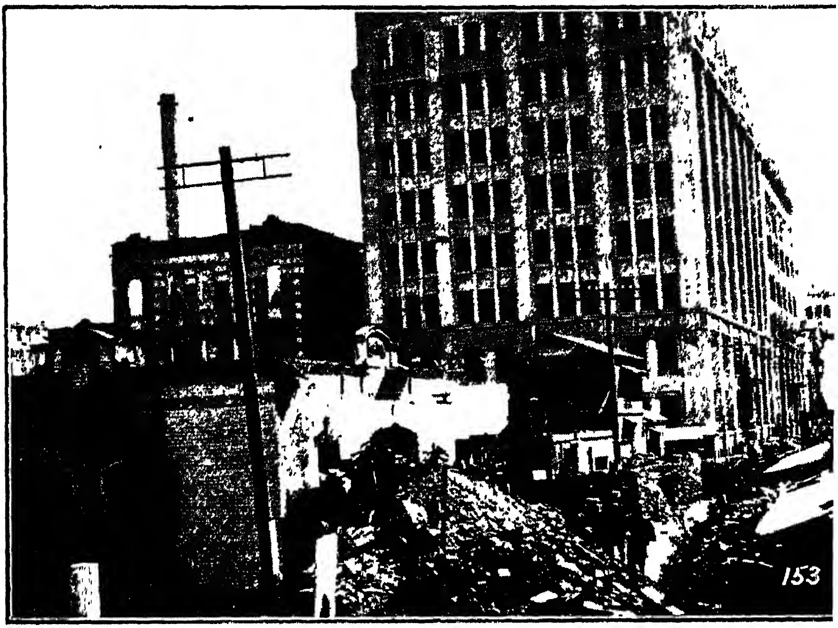


FIG. 3.—MITSUI BUILDING, TOKYO, JAPAN.

A reinforced concrete structure 110 feet high, gutted by fire but uninjured by the earthquake. A well designed structure; the stairways and elevator structures are stiffened by an adequate amount of reinforced concrete.

The débris in the foreground is from an adjoining brick building. The chimney stack on the left is of reinforced concrete it was not damaged by the earthquake.

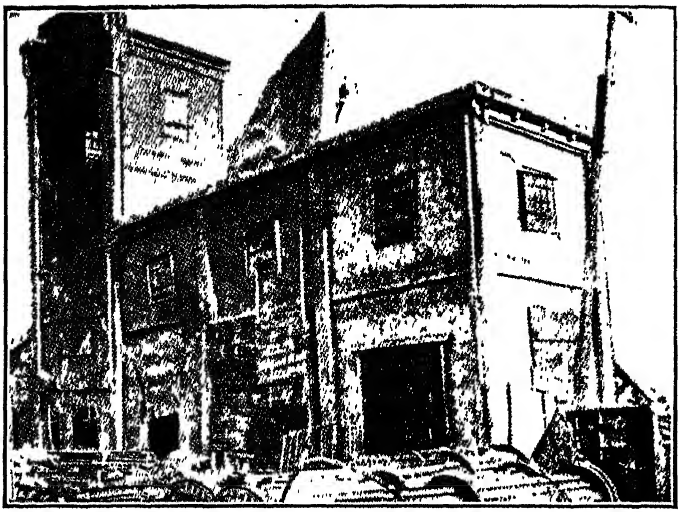


FIG. 4.—REINFORCED CONCRETE WAREHOUSE AT HIGASHIKANAGAWA, JAPAN.
Damage due to foundation settlement during the earthquake.

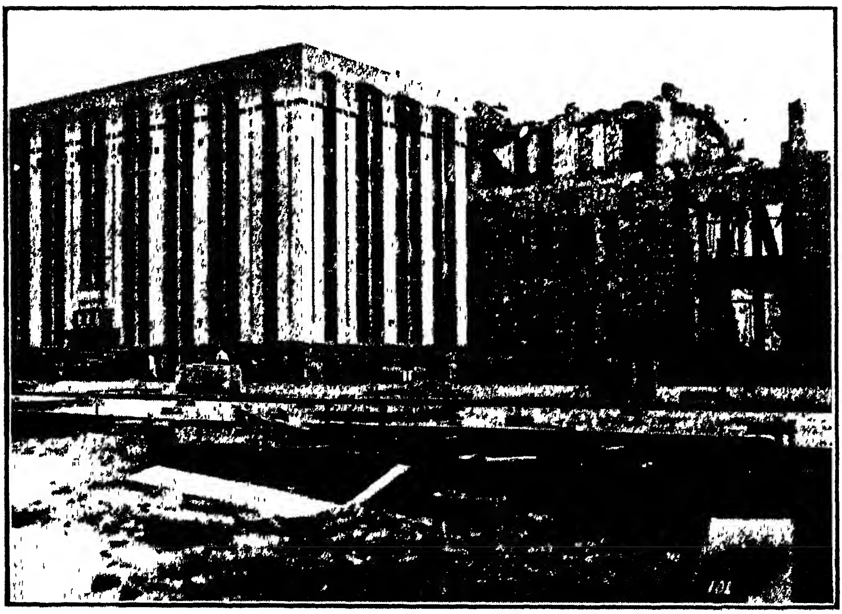


FIG. 5.—A 4½ STOREY REINFORCED CONCRETE BUILDING AT MARUNOCHI CENTRAL, TOKYO, JAPAN.
This building was absolutely undamaged by the earthquake while surrounding buildings of other materials were destroyed.

There were 16 structural steel frame buildings in Tokyo at the time of the earthquake, 6 of these were undamaged, while 10 sustained more or less damage. The 6 undamaged were those that had reinforced concrete filler walls, the others had brick wall construction, and sustained more or less damage due to the shattering of their external walls and breaking of interior partitions. Outside of Tokyo no large steel frame buildings existed in the earthquake region.

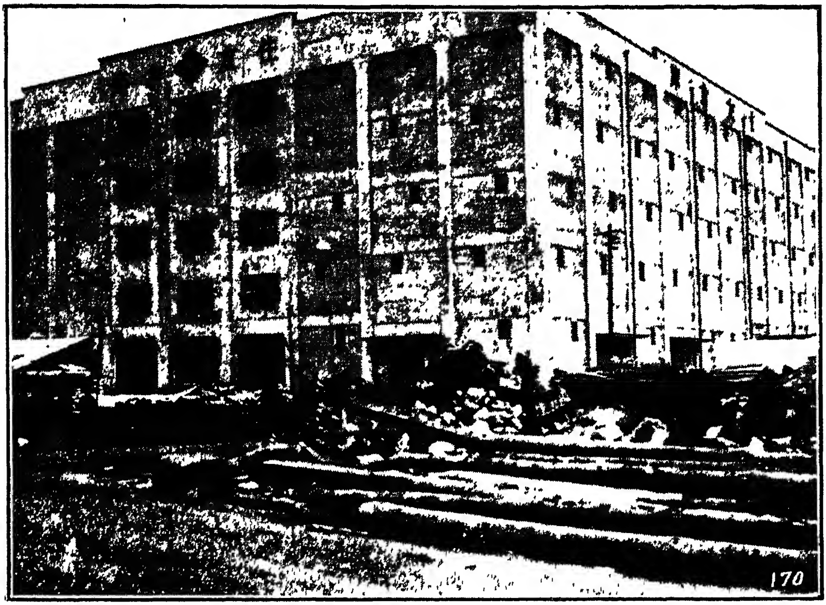


FIG. 6.—SUMITOME WAREHOUSE, TOKYO, JAPAN.

A splendid example of a reinforced concrete warehouse, 180 feet by 200 feet by 80 feet high. Note the reinforced concrete filler walls. At the time of the earthquake the building was loaded with goods valued at $4,000,000. It successfully withstood both the earthquake and fire.

Reinforced concrete had been used for buildings in Tokyo for about 15 years, and structural steel about 5 years; brick had been used extensively for many years. Buildings of brick construction, with a few splendid exceptions of good work, failed badly and were

responsible for great loss of life. Of 485 brick structures investigated in Tokyo, 49 of which were situated on the relatively firm higher ground, and 436 on low lying ground, the following was reported.

	Entirely Collapsed	Partially collapsed	Badly damaged	Slightly damaged	Un-damaged	Total
Hard Soil	3	9	15	16	6	49
Soft Soil	44	104	87	120	81	436
Totals	47	113	102	136	87	485

FIG. 7.—YURAKU BUILDING, TOKYO.

A steel frame structure with stoccoed brick facing and filler walls; damaged by the earthquake of 1923.

The steel frame buildings with reinforced concrete filler walls suffered no damage.

In the vast amount of shattered brickwork, fractures in general occurred through the brick itself, and not along the mortar joints; the inner mortar joints were shown to have been well filled and without voids: all

the workmanship was good. The weakness of this class of construction was shown to be its low tensile strength, which is insufficient to withstand the bending and swaying caused by an earthquake, unless the walls are of very considerable thickness and well stiffened by the floors and heavy division walls, and the whole well tied together.

FIG. 8.—WRECKAGE OF THE GRAND HOTEL, YOKOHAMA.
A Brick Structure destroyed by Earthquake.

In Yokohama nearly all of the buildings that remained after the earthquake and fire were of reinforced concrete.

In each instance of severe damage to reinforced concrete buildings it was determined that the damage was due to inadequate foundations or to the absence of proper wall bracing. The latter condition was particularly noticeable in factory buildings where there was practically no wall, the space between the columns and the beams at each floor being filled with glass.

A general survey of the situation in Japan led to the following conclusion :—The performance of reinforced concrete under the test of earthquake and fire can only be classed as highly satisfactory. Properly designed structural steel buildings, well braced and thoroughly fireproofed can be made earthquake proof, the simplest, cheapest and most efficacious bracing being secured by making the wall construction of reinforced concrete. The failure of reinforced concrete buildings was due to

FIG. 9.—WRECKAGE OF THE FRENCH CONSULATE BUILDING, YOKOHAMA

A Brick Structure destroyed by Earthquake.

Figs. 8 and 9, compared together with the illustrations of the other buildings in earthquake areas, show the superiority of reinforced concrete structures for resistance to earthquake or other severe vibration.

one or more of the following conditions :—(1) Inadequate foundations ; (2) Violation of commonly accepted principles of engineering design ; (3) Lack of rigidity in the buildings ; (4) The quality of the concrete, which was uniformly inferior.

Earthquake at Santa Barbara, California.—On June 29, 1925, a rather serious earthquake occurred in Santa Barbara, California. The author happened to be within the earthquake zone at the time, and experienced the shock ; he also made an inspection of the buildings in the devastated area a few days afterwards, and drew similar conclusions to those contained in the report on the Japanese disaster. The defects caused by the Santa Barbara earthquake, however, were more easily determined since the wrecked buildings were not

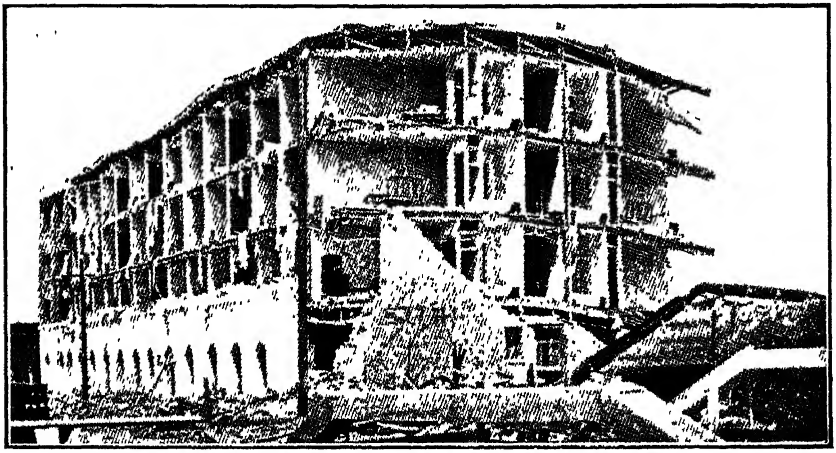


FIG. 10.—HOTEL CALIFORNIAN, SANTA BARBARA, CALIFORNIA.
Destroyed by earthquake. The main walls were of ordinary brick construction, the partition walls of wood studs. Destruction due to lack of rigidity of frame ; absence of structural bond and adequate ties. Inspected by the Author.

altered by subsequent fire. Reinforced concrete buildings which were properly designed and well tied together with reinforced concrete walls suffered very little or no damage. Badly bonded brickwork, hollow tile, cut stone and concrete block failed badly. The absence of a reasonable number of steel frame buildings prevented a comparison of these with other classes of construction.

As a result of the above inspections, and by a comparison of the illustrations, it can be concluded that reinforced concrete buildings, well designed and built

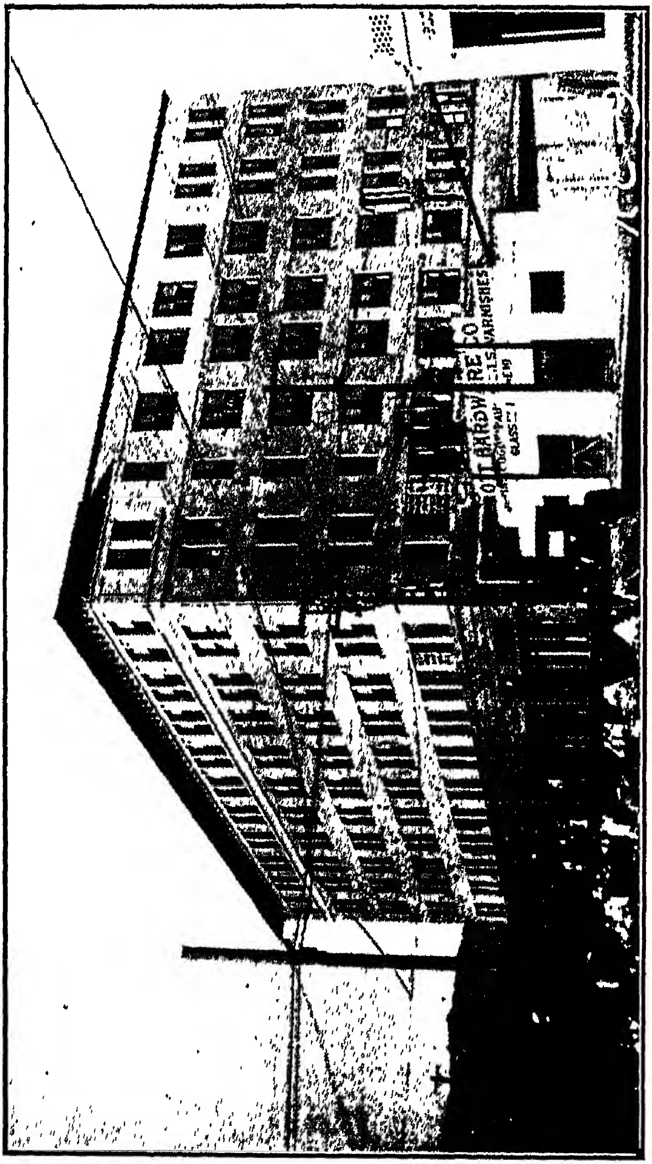


FIG. 11.—CENTRAL BUILDING, SANTA BARBARA, CALIFORNIA.

A 6-storey reinforced concrete frame building with clay tile filler walls. Situated close to the building of Fig. 10. The concrete frame suffered no damage by the earthquake, but the filler walls were cracked in a number of places, due to the swaying of the structure. Note the exposed clay tile of the second and third stories. Inspected by the Author.

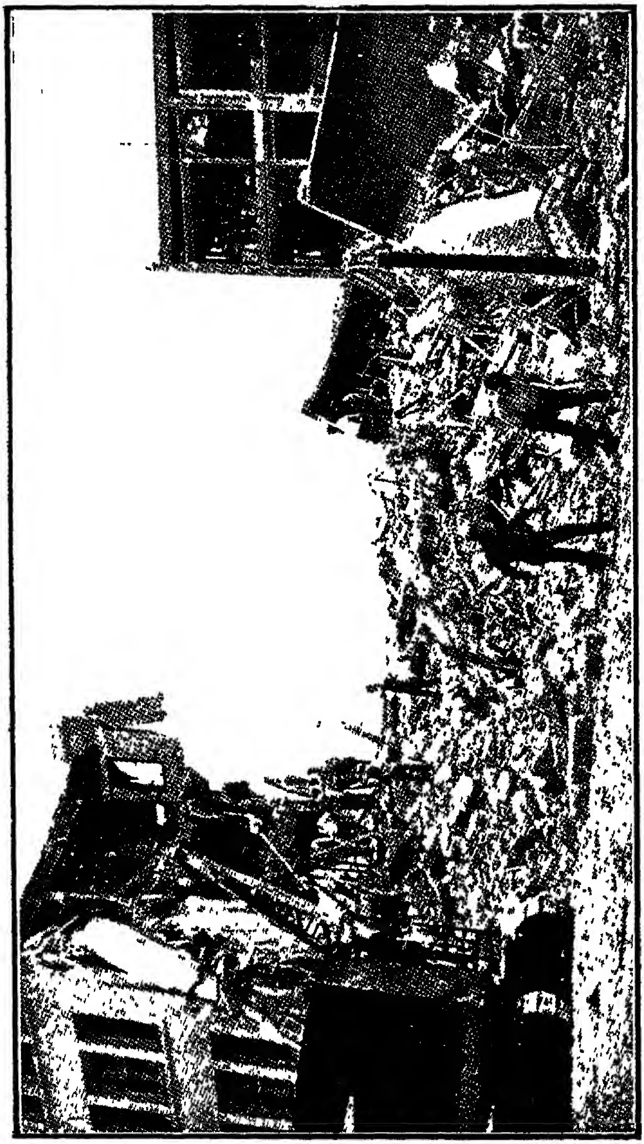


FIG. 12.—SAN MARCOS BUILDING, SANTA BARBARA, CALIFORNIA.

A reinforced concrete office structure, destroyed by earthquake through poor design, materials and workmanship. Inspected by the Author.

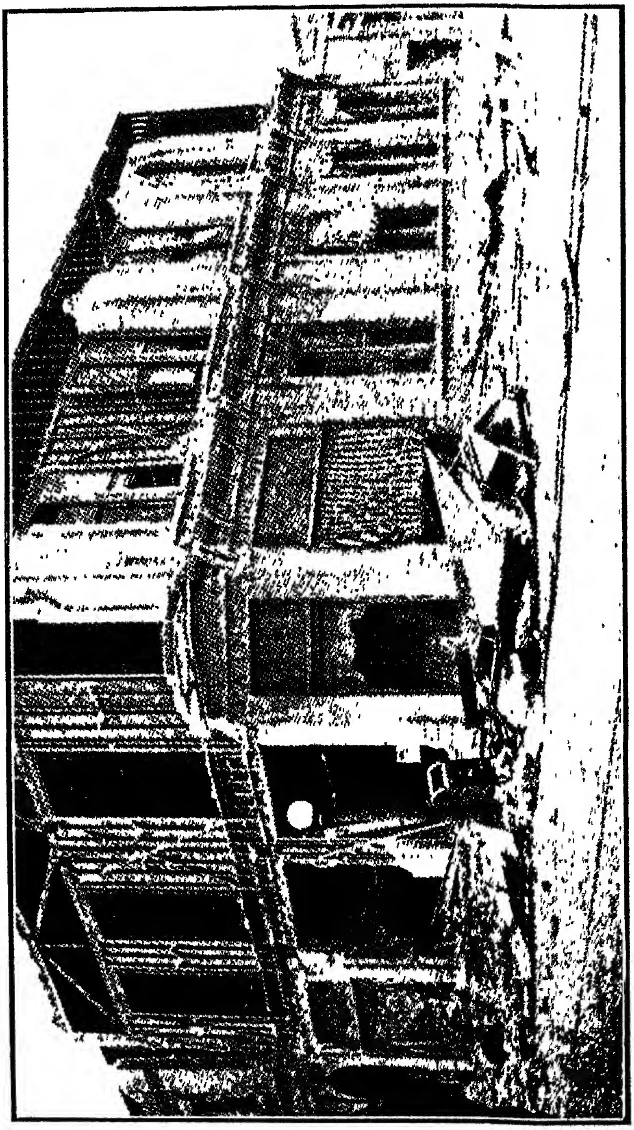


Fig. 13.—American Legion Building, Santa Barbara, California.

One of many brick veneer buildings destroyed by the earthquake of 1925, showing the inadequacy of this class of construction for earthquake resistance. Inspected by the Author.

are highly satisfactory in resisting earthquakes. The following points, however, require careful attention :—

(1) Adequate foundations, firm and unyielding.

(2) Provision to secure perfect rigidity of the whole structure when subjected to horizontal motion and reversal of stress.

(3) Walls between columns and beams must be reinforced concrete, and well bonded to the columns and beams, so that the whole acts as one ; brick and hollow tile panels should not be used.

(4) Walls and columns must be securely anchored to foundations. All footing studs and column verticals should be hooked at the splices, the laps being designed for tension, not compression. Adequate reinforcement must be used throughout the whole structure.

(5) Outside walls must be well tied to the inner walls and the floors so that there will be no relative movement of one with respect to the other. Corners of window openings require diagonal bracing bars or additional well-anchored vertical and horizontal reinforcement.

(6) All parts of the building must be so bonded together and braced laterally as well as vertically, so that the entire structure will tend to sway as a unit.

(7) All work joints must be kept free from laitance, rubbish and dirt.

HURRICANE RESISTANCE.

The advantages of reinforced concrete in resisting strong winds was shown in the great Florida disaster in September, 1926. A careful examination of all the reinforced concrete buildings in the business section of Miami revealed no structural damage of any kind, and in no case was there visible evidence that the buildings had moved in any direction, nor was there any cracked plaster due to, or indicating, any movement. The entire effect of the hurricane was confined to the breaking of windows and destruction of awnings. E. C. Romfh, Mayor of Miami, reported that it was remarkable that a city the size of Miami should have gone through such a severe storm with comparatively so

small a number of dead and injured. This he accounted for by the fact that the city had the largest percentage of concrete buildings of any city in the United States.

The wreckage of other buildings in the storm area showed the necessity of bracing in walls, stiffening of high parapets, thorough tying of roofs and floors to walls, and the use of good materials with first class workmanship. Good standard materials properly used suffered no discredit from the hurricane. Good materials with inferior workmanship, or poor materials substituted in the boom period when adequate inspection was impossible, spelled disaster wherever the storm struck.

FIRE RESISTANCE.

The superior resistance of reinforced concrete to fire is due to the steel being protected from intense heat by the concrete, which is found to be one of the most suitable materials for this purpose, it being a very bad conductor of heat ; also the expansion of concrete and of steel under normal temperatures is practically equal, the co-efficients per degree of temperature being 0·000,006 and 0·000,006,5 respectively ; this means that with an increase of heat sufficiently to raise the temperature of the concrete and the embedded steel 500 degrees, in a length of 20 feet the steel would expand $20 \times 12 \times 0{\cdot}000{,}006{,}5 \times 500 = 0{\cdot}78$ inch, and the concrete $20 \times 12 \times 0{\cdot}000{,}006 \times 500 = 0{\cdot}72$ inch ; consequently, with the poor heat conducting property of the concrete, and well designed and constructed work, it is impossible for the steel to absorb sufficient heat to cause it to expand enough in excess of the concrete to cause fractures, or for the two to absorb sufficient to expand together enough to seriously endanger the structure from this cause. This danger is still further reduced by the steel being comparatively small in volume to that required in ordinary steel construction.

Probably the most thorough investigation that has ever been made of the effect of an intense fire upon reinforced concrete structures is that following the

great fire at the plant of Thomas A. Edison, Inc., at West Orange, N.J., on December 9, 1914. Evidences of fused steel and even concrete were reported, showing that the temperature in some of the buildings must have reached as high as 2,000 degrees F., and probably exceeded 2,500 degrees F. It was considered that owing to the scarcity of water and to the abundance of highly inflammable materials, this excessive temperature con-

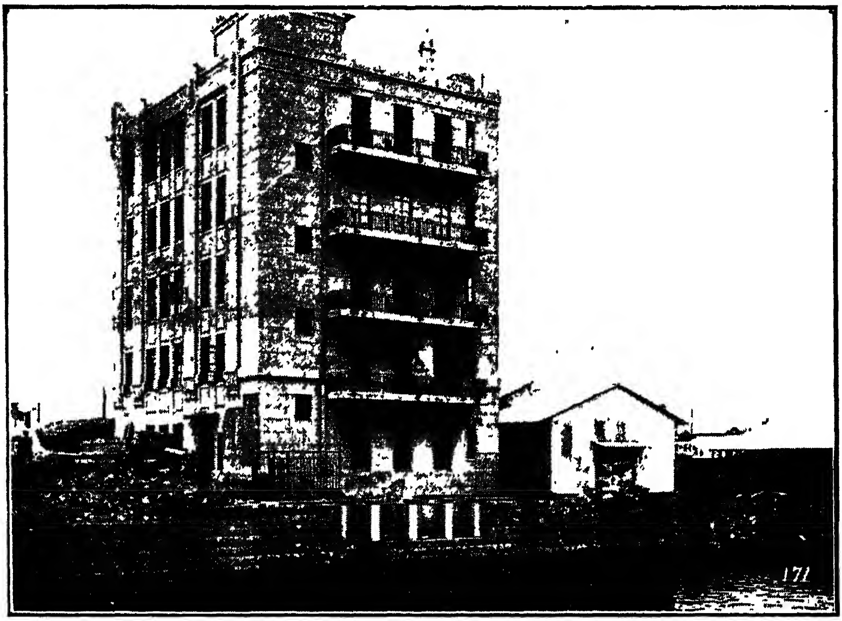


FIG. 14. (*See Fig.* 15.)

tinued in places for several hours. The joint report of the National Board of Fire Underwriters and the National Fire Protection Association, stated the severe loss to be due to lack of adequate protective measures in the matter of fire walls, fire resisting door and window construction, automatic sprinklers and water supply. It further stated that the concrete panel walls withstood the fire excellently. There was no case of individual collapse, either of a floor slab, beam or girder, except in one basement where the concrete and reinforcement

were apparently melted away, causing failure of three beams and a floor slab. All other failures were stated to be due to the columns, chiefly through longitudinal floor expansion, except in the case of various wall columns of which the failure was attributed to the great difference in temperature between the inside and outside faces of the wall, and to their rigid position vertically, which made them less able to resist the building expansion.

FIG. 15.

Figs. 14 and 15 are from photographs of a small Apartment Building and a Commercial Building in Tokyo, Japan, both of reinforced concrete construction; they are good examples of the splendid fire resistance of this class of construction. Both of these buildings were gutted by the fire following the earthquake in 1923. The structural parts sustained no damage whatever by either the earthquake or the terrific heat of the subsequent fire.

The corners of square columns spalled off where circular columns suffered very minor damage. Although the columns appear to have suffered most, it was pointed out that more remained intact than were seriously injured. The report concluded that a reinforced concrete building

could doubtlessly be built that would satisfactorily withstand such a fire. In the American Concrete Institute's report on this fire, it was stated that the fire fully demonstrated the advantages of monolithic structures. The fact that at five different places several of the wall columns were rendered useless, and yet the upper portions of the building stood intact, is evidence of the superior merits of concrete in monolithic construction. The end walls in three upper floors of two buildings extended above the roofs of the adjoining buildings which were completely destroyed; while this was in the hottest part of the fire, the walls were practically undamaged, which was an admirable demonstration of the value of concrete walls as a fire barrier.

Thomas A. Edison stated that the report of his engineers showed that 87 per cent. of the reinforced concrete buildings, which were subjected to a very intense heat remained in good condition, and of the machinery which they contained, about 85 per cent. could be again used with small repairs. Buildings of other materials, together with their contents, were entirely destroyed.

CARE IN DESIGNING AND CONSTRUCTION.

To secure the valuable properties of reinforced concrete, the work must be properly constructed as well as designed. The former is quite as important as the latter, for if the greatest of care is taken with the design and careless supervision given to the construction, the result may be as bad or worse than if the work was badly designed; in fact, a poor design well constructed may give a better result than a good design badly constructed. The supervision should include:—Careful inspection and testing of all the materials; attention to the preparation, erection and removal of the forms, to the gauge mixing and placing of the concrete, to the size and placing of the steel according to the drawings, to the position and condition of slabs and beams where left unfinished at the end of a day's work, to the condition of unfinished work before its continua-

tion or completion, and to the protection of newly finished work from building operations and inclement weather. All these points are of the utmost importance if we aim at the best possible results, for if the materials are not carefully selected and the workmanship is not good, the result cannot be satisfactory. It is therefore necessary that constant expert supervision be given to the construction from its beginning to its end.

Only men experienced in this class of work should be engaged for the purpose, and under no consideration should an inexperienced man be placed in charge. The importance of this will be understood by any practical man who has observed the general carelessness with which Portland cement concrete work is often executed. Some workmen still have the idea that anything will do for concrete, both as regards the material and the mixing, and it is a material that a dishonest contractor can easily manipulate to his own advantage. The strength of the work is also greatly diminished by the irregular manner in which it is often deposited and tamped.

Although the designer may carefully specify and be satisfied with the quality of the cement and aggregate, the work may fail through the materials not being gauged exactly in the proportions arranged for when designing, through insufficient or irregular mixing, by an excess or insufficiency of water, or through badly placing and tamping. All these details require very strict supervision ; it is absolutely necessary for each operation to be carefully and perfectly done, otherwise the strength and durability will be very seriously impaired.

Skilled labour should also be employed for the construction, erection and removal of the forms and supports, for many cases of collapse have been due to defective form construction, or careless removal. The arrangement and fitting together of various parts of the form work demand much consideration and skill, for all the parts must be fitted together and supported in such a manner as to secure rigidity, strength to resist the tamping and weight of the wet concrete and vibration

caused by building operations, without alteration of form, also to secure ease of removal, to avoid the use of heavy hammers, levers or force which might disturb the work. It must, however, be borne in mind that form work is only temporary ; consequently, waste of material and unnecessary labour in the preparation and fixing of forms must be avoided. The lumber is best if well naturally seasoned, for green wood is apt to shrink and warp after being placed in position, and will consequently leave a rough uneven surface to the work ; kiln dried lumber will expand and buckle with the wet concrete. No complicated joints are necessary ; all should be as simple as possible, such as the butt, lapped, and fish varieties, with as few nails as possible, but with a frequent use of folding wedges and wood clamps.

Wherever possible struts and studs should be in one piece, but where these are long it is sometimes convenient to make them of two pieces which can be done as shown by Fig. 16, the strips being nailed on all four sides. The butt joint should be cut square and true.

All studs should have a firm footing which can be provided for as shown by Fig. 17 or Fig. 18 ; the folding wedges admit of easy adjustment and removal. Fig. 19 illustrates suitable details for the head of a stud supporting a beam form ; it also shows how the bearers under the boards which support the floor slabs can be supported by utilizing the ledges of the beam forms. Fig. 20 illustrates the connection of secondary beam forms to main beam forms.

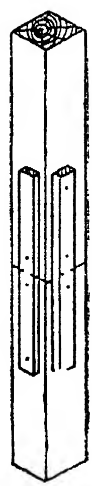


Fig. 16.

Columns forms may be as Fig. 21, which also illustrates the connection of beam forms to column forms. Instead of the wooden clamps, as illustrated, there are several kinds of metal clamps now made, some of which are patented. Of course any of these can be used providing they are of sufficient length or are placed sufficiently close together to prevent bulging of the forms Fig. 22 is a suggestion for large columns, the cross pieces being loosely cut in between the side pieces

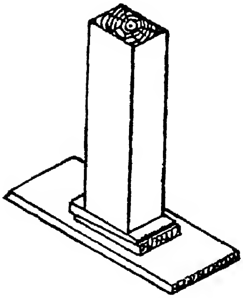


FIG. 17.

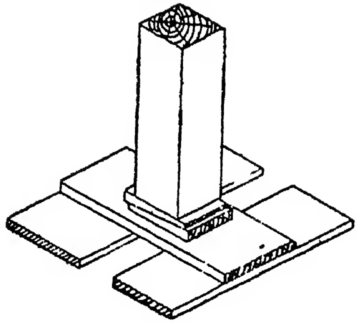


FIG. 18.

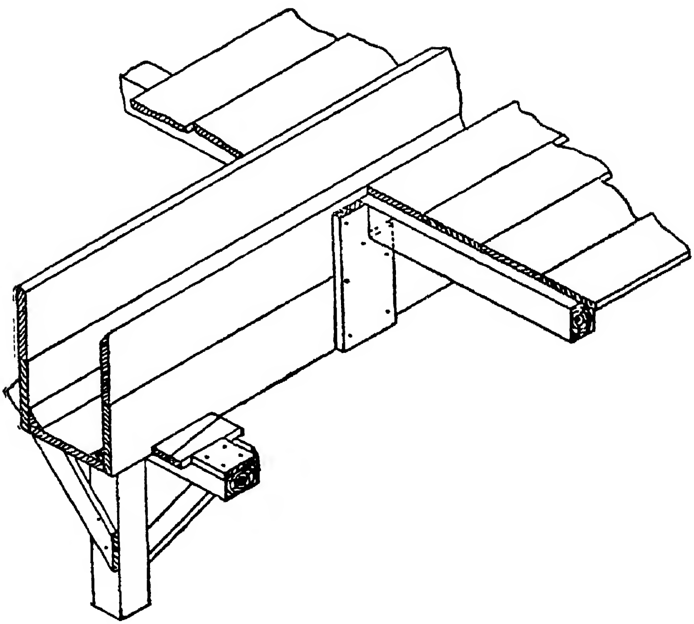


FIG. 19.

and held in position by the bolt passing through two staples, as indicated; these pieces are far enough apart to allow wedges to be inserted, as illustrated, otherwise there would be a difficulty in arranging the position of the bolts to allow the cross pieces to fit tightly against the forms. In all form work braces and horizontal ties

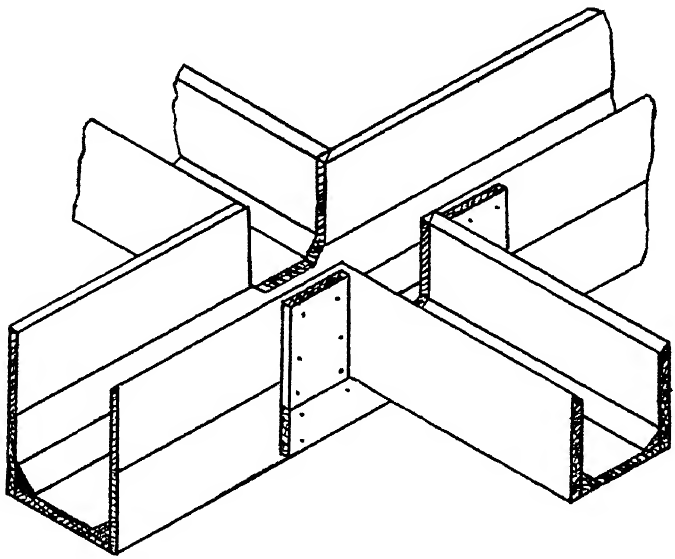


FIG. 20.

should be freely used to guard against displacement by blows from traffic or vibration from building operations.

Walls forms should be well supported and have a plentiful supply of distance pieces, tie wires or bolts, and struts.

For extensive work, particularly where there is considerable detail, and for curved work, steel forms should be used. There are manufacturers who specialize in these and supply them for any shape or size of work, made in convenient size knock-down sections, with connections designed to enable the forms to be easily fixed or removed without transmitting any shock to the

newly constructed work. These take less time to erect and remove, are more rigid, and will leave the concrete cleaner than wood forms. The initial cost is greater than wood ; consequently, for small jobs with little repetition

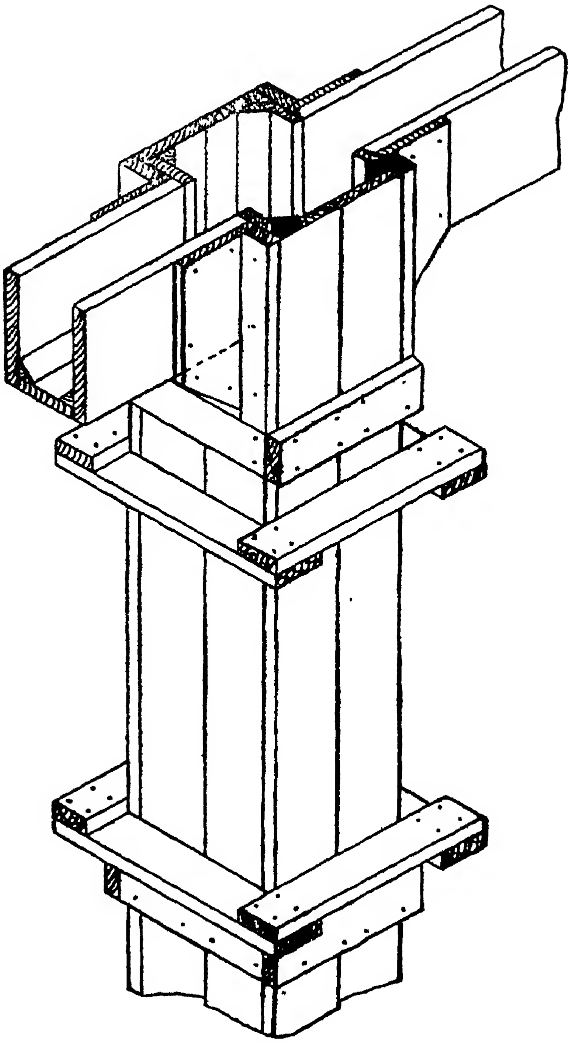


Fig. 21.

they are not economical, but for such work as above described there is considerable saving in cost, and gain in time and in quality of work.

Adjustable steel forms for floor work, and their supports, are obtainable as illustrated by Figs. 23, 24 25 and 26.

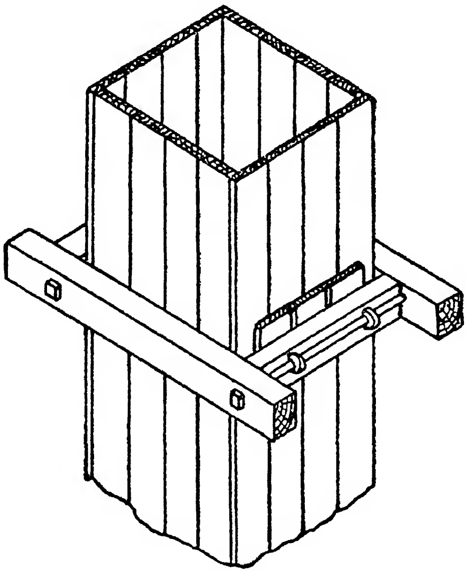


Fig. 22.

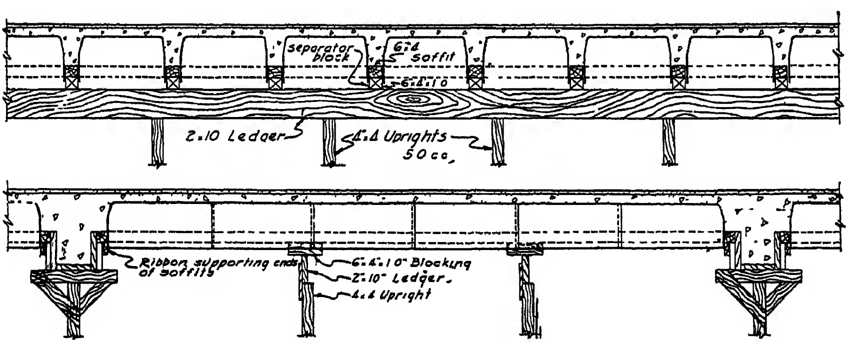
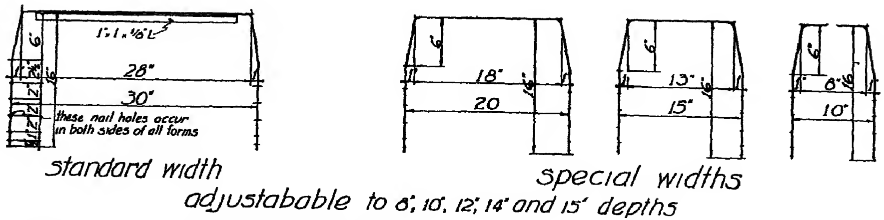


Fig. 23.—Adjustable Steel Forms for Rib Floors



30" wide *Standard* Width Intermediate steelforms are furnished in 1 2 and 3-foot lengths.

20", 15", and 10" wide steelforms are *Special* Width Steelforms The intermediates are furnished in 1 and 3-foot lengths

FIG. 24.—ADJUSTABLE STEEL FORMS AND THE FALSE WORK IN POSITION, FOR A FLOOR PANEL 20 FEET SQUARE.

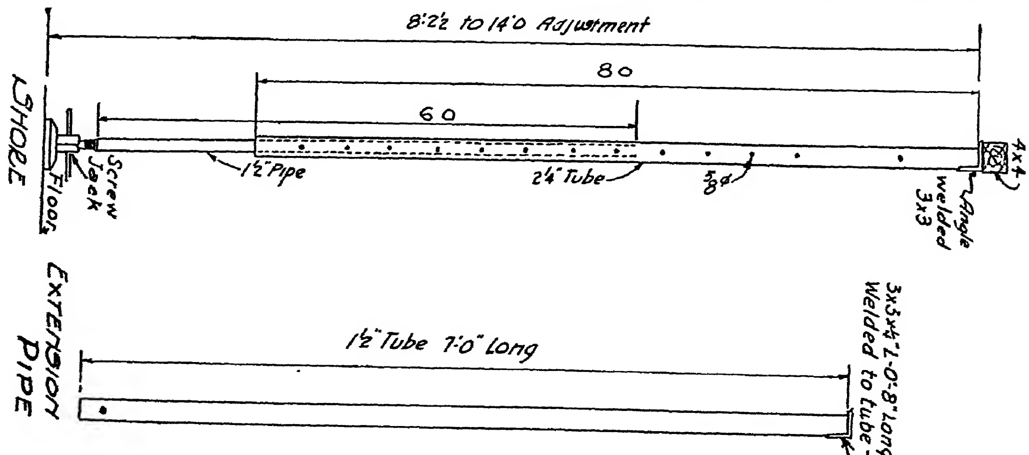


FIG. 25.—ADJUSTABLE STEEL SHORE FOR SUPPORTING FLOOR FORMS.

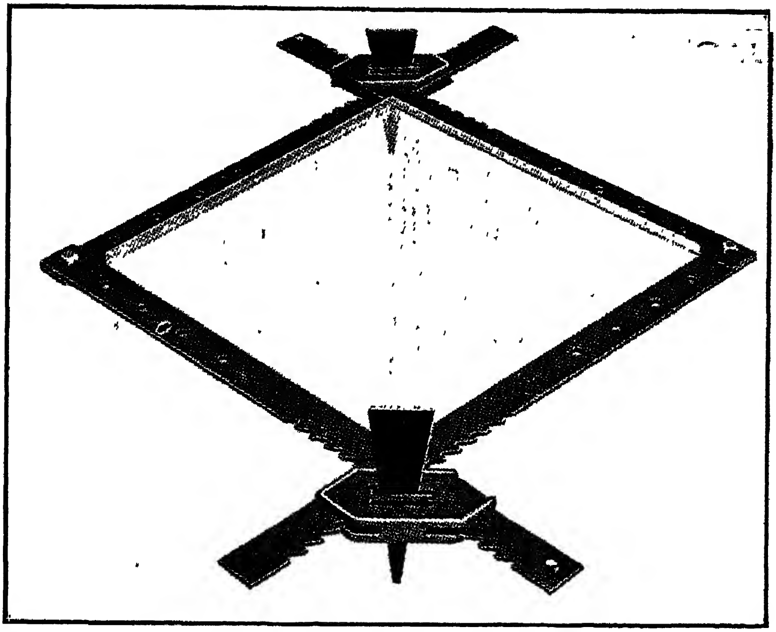


FIG. 26.—A PATENT ADJUSTABLE COLUMN CLAMP.

TIME TO REMOVE FORMS.

The time to elapse before the supports and forms may be removed will depend upon the weather and on the position and nature of the work, as well as upon the setting property of the cement. Under ordinary conditions, for small slabs and columns they can be removed in a week, but if traffic or building operations are to be continued on or about the work they should be left much longer. None of the forms of heavy beams and arches should be removed until at least two weeks have elapsed, the soffit supports should be left for at least four weeks. See cause of floor collapse, page 36.

MIXING AND DEPOSITING.

Concrete is now very rarely mixed by hand, but if it is necessary to do this it must in no case be mixed on the bare ground, but on a level platform of boards placed close together. The materials must be accurately measured to the gauge specified and thoroughly mixed, for which at least two men should be employed. The sand should be spread to a uniform depth and covered with the cement, then well mixed while dry by turning it over at least twice, after which the coarse material should be added and the whole turned over twice more while dry; then the water should be sprayed on while it is being turned over again, after which it should be turned over at least once more.

Not more than half a cubic yard should be mixed at a time, and it should be immediately placed in position; any left over and allowed to set must not be knocked up again for use other than as coarse material. The quantity of water required will vary according to the size, porosity and dryness of the coarse material; for instance, broken brick will require more than gravel, and if damp either will require more than if dry; consequently, with these conditions together with the practical impossibility of preventing irregular waste in applying, any specified quantity can only be an approximation; the following, however, are fair averages for different proportions of well graded aggregates up to 1½ inch gauge, and may be used as a basis for trial mixtures. For a 1-2-4 mixture about 6 gallons per cubic foot of cement; for 1-2-3 mixture about 5½ gallons, and for a 1-1½-3 mixture 5¼ gallons; but the actual amount only can be determined when mixing; an experienced man can judge by the appearance, and feel while turning it over; if, however, a handful of concrete is squeezed and when released just retains its form without appearing sloppy, and does not break by adhering to the hand, it may be considered as in a suitable condition. An

excess of water will increase the tendency of the concrete to crack in shrinking, and will make it more susceptable to the formation of laitance seams ; it will also weaken the concrete and retard its setting.

For irregular work and work containing a multiplicity of rods in which it is difficult to obtain free access to all parts for tamping, many engineers specify a wet mixture ; this is a mixture containing more water than that specified above ; the additional water assists consolidation and less tamping is required, but the concrete does not attain so high an ultimate strength as a drier mixture well tamped ; otherwise there is no objection to this method providing too much water is not used and tamping is not entirely dispensed with. With an excess of water and hand mixing there is danger of the cement being more or less washed off the aggregate. The author has often seen so much used in this method of mixing as to wash a considerable portion of the cement to the bottom of the heap where it has been shovelled up like slurry, and in some cases of either hand or machine mixing, sufficient to flood the surface of the concrete after it is deposited in place. It is also very usual to see so much water used that in pouring the slabs the workmen are able to walk through it almost as easily as through water. Such carelessness can only result in inferior work and should be strongly condemned.

For any other than a small job mechanical mixers should be used, they are generally operated by steam or gasoline power ; this method will produce a concrete far superior to that of hand labour, for by it a more uniform mixture can be obtained and a great saving of time effected. The whole mass of concrete should be in continuous motion within the machine, after the water has been added, for a period of not less than one minute, and for not more than two minutes. The water should not be added until after the dry materials have revolved several times in the mixer. The chief risk to guard against is the use of an excess of water, for a first class mix will not leave the hopper very readily ; the more water the easier will the concrete be emptied from mixer

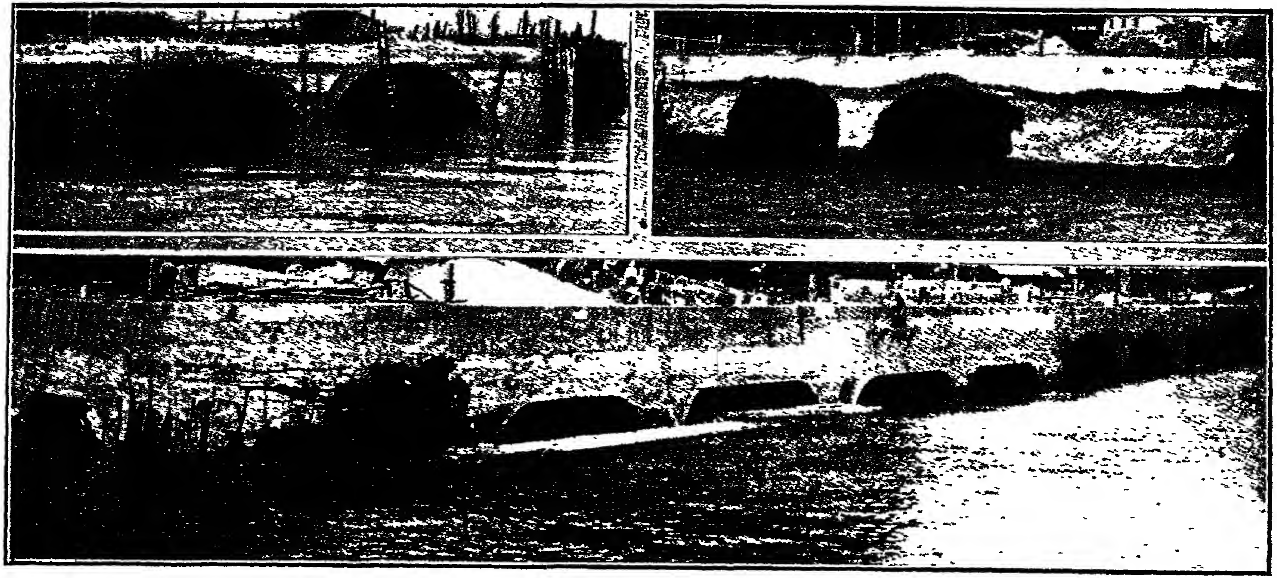


FIG. 27.—EFFECT OF CONSISTENCY ON WEAR OF A CONCRETE PIER.

The upper left-hand view, taken just after construction, is the end of a pier on the New England coast: the concrete was of a very dry consistency. The upper right-hand view shows the same portion of the pier as that on the left, after 8 years of service, very badly eroded.

In the lower view, the concrete of the left-hand portion was of a very wet consistency, and is also badly eroded; the portion on the right was of a medium consistency, and found to be in excellent condition after 20 years of service.

Courtesy Engineering News-Record.

and barrow or bucket ; consequently constant attention is necessary to prevent an excess being used, as speed is generally the first consideration with the contractor.

Some engineers specify that the quantity of water is to be determined by the slump test on the job. This is done by means of a metal form the shape of the frustrum of a cone, 12 inches high, 4 inches in diameter at the top, and 8 inches at the bottom. This form is filled with the concrete to be tested, which is well tamped with a pointed iron rod while it is being placed ; when filled the form is immediately lifted off, and the slump or settlement measured. The following slumps are considered to be the most suitable :—

(1) For concrete roads, from ½ inch to 1 inch.

(2) For foundations and mass work, from 1 inch to 1½ inch.

(3) For reinforced concrete work, from 2 inches to 2½ inches.

For reinforced concrete work in which the bars are very close, or where a greater plasticity than that of a 2½ inch slump is desirable, probably on account of the difficulty of tamping, a greater slump is specified, 4 inches being quite common ; but in no case should more than 6 inches be allowed ; for the greater the slump the greater the loss in the compressive strength of the concrete.

Slabs and beams should be commenced and completed the same day ; if, however, this is not possible, the concrete should be filled the entire depth, and left unfinished with a vertical face over the centre of a support or at the center of the span. In these positions the diagonal tension and shear, which the concrete is designed to resist, is less than elsewhere ; consequently an imperfect joint here would be less harmful than it would be in any other position. When recommencing the work the unfinished surface should be well watered and given a coat of grout, or thin cement mortar immediately before the fresh concrete is deposited against it ; if the work has been left several days, is very dry and dusty, a portion of the surface should be broken away and the waste well swept up before it is watered and grouted.

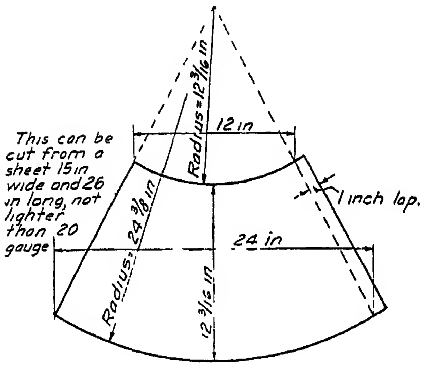


FIG. 28.

PATTERN FOR MAKING THE FORM TO BE USED IN THE SLUMP TEST

FIG. 29.

MAKING THE SLUMPS.

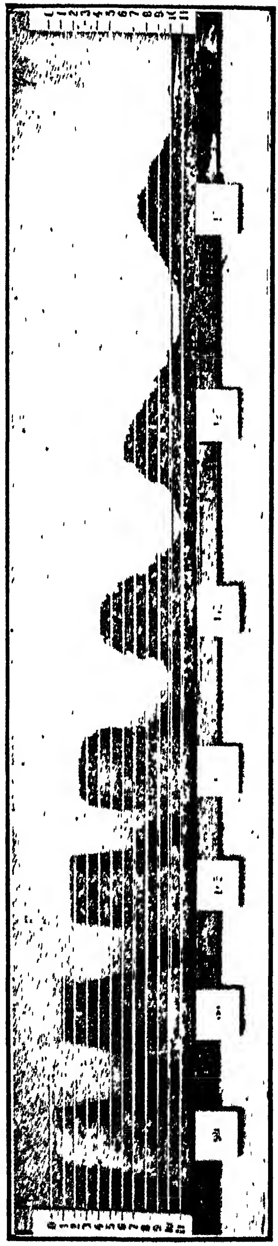


FIG. 30.—ILLUSTRATING VARIOUS SLUMPS.

Exposed new surfaces should be protected from traffic, frost, heavy rain, and the sun. During hot weather it should be kept damp for a week ; during continuous frosty weather the work should be suspended, but for occasional light frosts it may be protected at night by a covering of tarpaulins, sacks, sawdust, wood-shavings, straw or boards. There is, however, grave danger in permitting concrete work to be done in a freezing temperature, whether continuous or intermittent, although this is the case, important work is often carried on during very severe winter temperatures, notably in the northern parts of the United States and in Canada, where in some parts the temperature remains many degrees below zero for several months each year, with freezing weather from October to April. These conditions make the building season very short ; consequently, all kinds of methods are resorted to, with their attendant risks, to enable concreting to be continued as far as possible into, or all through, the winter. In doing this, the object to be borne in mind is that the material when used must not be covered with ice crystals or contain frost, and that means must be provided to prevent the concrete from freezing after it is placed in position, and until it has thoroughly hardened. The usual method adopted for endeavouring to secure the desired result is to heat the aggregate to well above freezing point by means of burying within the aggregate pile, pipes through which steam is forced from the concrete mixer or some other heater ; sometimes the aggregate is piled up over large iron cylinders within which fires are kept burning. No part of the aggregate should be heated to more than 150 degrees F. The mixing water should also be heated. The addition of common salt to the water will lower the freezing point of the concrete, but there is a limit to the quantity that may be used. See information on "Salt Water Concrete." The concrete when placed should have a temperature of from 75 to 80 degrees. F. Metal forms should be warmed before placing the concrete ; this can be done by a jet of steam or by wetting with hot water.

After the concrete is placed in position it must be

well covered over and further protected wherever possible by hanging wind sheets or hoarding. In addition to these means, movable stoves or salamanders may be used in enclosed structures. Many extensive jobs have been completed, apparently successfully; by these means, such work, however, is extremely risky.

Before removing the forms, be sure the concrete is hardened, not frozen. If there is doubt in this respect remove a small portion of a form, and apply a jet of steam or a little hot water to the exposed concrete. If frozen, the heat will soften the concrete by thawing the water contained in it.

It is well known that the freezing of green concrete retards the setting and greatly reduces its strength; many cases of collapses have been traced to this cause. A particular case came under the author's notice in November, 1915, in a city in the state of Illinois, U.S.A., in which the collapse of the floors of a new factory building caused the death of two men. The floor spans were 17 feet by 21 feet, the slabs were 7 inches thick, and the drop panels around the column heads were 11 inches thick. The first floor had been poured about six weeks during which time there had been some very cold weather. The second floor was poured on the day previous to the accident. The day of the collapse was very wet. At the time of the accident the forms under the first floor were being removed. The first indication of failure noticed was a crack through one of the column heads; then the floor collapsed and the forms and green concrete of the upper floor fell with it, stripping clear of the reinforcement. The column heads of the first floor were sheared off. Expert engineers testified before the coroner's jury that there was considerable frost in the concrete slab, and that the concrete was very porous, owing to the use of an excess of water in order to make the concrete flow more readily around the reinforcing rods, and that there was an insufficiency of supports to hold the slab rigidly during the period of setting. The proportions were good, but in pouring the concrete the heavier gravel separated from the sand and cement. Sam-

ples tested at the laboratory of the University of Illinois showed insufficient strength. The opinion given by the engineers was that the accident was due to premature removal of the falsework supporting the forms, the freezing weather having retarded the setting of the concrete.

The coarse material to be used for the concrete should be clean and free from foreign matter, and carefully proportioned with a view to securing a maximum density.

The quantity of sand required will depend upon the nature of the coarse material ; sufficient must be used to ensure all the voids being properly filled. It is better to have a little in excess rather than an insufficiency. It should be borne in mind that density and imperviousness are absolutely necessary to secure good work, everything possible should be done to obtain this result. The sand should be dry, for if wet it is impossible to thoroughly incorporate the cement. For good work, where strength is the primary consideration, the most usual mixture is four parts by volume of broken granite or trap rock, hard crystaline limestone, sandstone, gravel, or selected slag sufficiently small to pass a $\frac{3}{4}$ inch mesh, two parts of sharp coarse sand, and one part of Portland cement, known as a 1-2-4 gauge. These proportions and size of aggregate, however, should not be taken as a standard, but before designing important work the materials for the concrete should be selected according to local circumstances ; then varying mixtures should be prepared, the proportions being carefully tabulated, and the test pieces made by tamping the concrete, as described for the work, into square or cylindrical moulds not smaller than 6in. × 6in. × 6in., and at the end of twenty-eight days four pieces of each variety should be tested by compression with a gradually applied load, and the average results of each set compared, the most satisfactory variety being selected for use and the work designed and the gauge specified accordingly.

The size of the aggregate is often specified as $\frac{1}{2}$ inch for light slabs or partitions, where expanded metal or

mesh is used, $\frac{3}{4}$ inch for flat slab floors, beams and slabs, girders, columns, footings, retaining walls and other moderately heavy work, and $1\frac{1}{2}$ inch for heavy work.

With the hard materials gauged 1-2-4, the crushing strength at 28 days should be from 2,400lb. to 3,000lb. per square inch. The working stress for beams should not exceed one-fourth, and for columns one-fifth, of the crushing resistance at 28 days ; consequently, the working stress for the hard class of materials is usually taken as 600 lb. per square inch for beams, and 500 lb. per square inch for columns ; it must, however, be borne in mind that the allowable amount should be determined by experiment, as described above. The building ordinances of some cities specify the maximum stress allowable in the district governed by that particular ordinance.

The coarse materials should be screened and all that passes a 3/16th-in. mesh should be considered sand.

Broken brick, furnace clinker, coke breeze, and limestone of average density are also used in districts where they are easily obtainable ; the ultimate strength of the concrete made from these, however, is only about half that of the harder materials if gauged in the same proportions ; they are also more porous, and should be well damped before being added to the cement and sand. More water is also required for mixing, and the concrete must be deposited in a wetter condition than that of harder aggregates or it will be difficult to tamp it sufficiently for securing the density necessary to protect the reinforcement from atmospheric influence and moisture, which protection is absolutely necessary, otherwise the durability of the structure will be limited according to the rate of oxidation of the steel. If, however, the concrete is practically air and damp-proof, oxidation will not take place ; it is a well-known fact that cement acts as a preservative from rust ; furthermore if steel bars are covered with rust when embedded in the concrete, the oxidation is immediately stopped. Moisture also considerably reduces the value of the adhesion of the concrete to the steel, which adhesion is a valuable factor for consideration when designing the work, par-

ticularly that in which the shearing stresses are high. The density of the concrete is also necessary as a protection from fire, for steel when exposed to intense heat, as in a conflagration, will expand and diminish in strength sufficiently to cause collapse of the structure. See page 5.

Although coke breeze, clinker, and broken brick are classed as second rate materials for strength, they are more fire resisting than those given as first-class materials, excepting the slag.

Slabs composed of coke-breeze concrete, after being raised to a red heat and quenched with water, have been found to be diminished in strength about 30 per cent., and only superficially injured and not past repair, but owing to Portland cement concrete increasing in strength with age, this class after being exposed to such conditions and having diminished in strength, will still be strong enough to carry the original working load. Brick concrete under the same conditions loses about 50 per cent. of its strength, and is damaged to a greater extent than coke-breeze, but not so badly as granite, limestone, sandstone and gravel concrete, all of which have been found to have suffered sufficient loss of strength and damage to necessitate renewal. Clinker and slag may be classed between breeze and brick in this respect ; consequently where fire resistance and strength are equal considerations, slag will give the greatest satisfaction, but for fire resistance, where great strength is not required, coke-breeze comes first, with clinker and cinder next ; great care, however, is required in the selection of clinker and cinder as they are likely to contain much refuse, which might be in sufficient quantity to entirely destroy the strength of the concrete ; the possible presence of sulphur is also to be feared as this will cause corrosion of the steel, and thus reduce its strength ; consequently, these materials should be looked upon with suspicion and treated with caution, and must not be used where strength is an important consideration.

The sand should be perfectly free from clay, vegetable loam, oil, animal or other foreign matter, and should be

sharp, gritty, hard silicious material having particles graded from fine to coarse, and of such a size that all will pass through a 3/16-in. mesh, and at least 75 per cent. pass a $\frac{1}{8}$-in. mesh. Fine sand alone will require more cement to produce concrete of the same strength as that obtained with sand graded from fine to coarse.

CHUTING CONCRETE.

For extensive works the most general method of depositing concrete is by pouring it through a chute, called spouting or chuting. This is done by means of a long sheet-iron chute attached to the top of a tower which is erected close to the mixer and material bins. The chute is made up of sections 20 to 40 feet in length; these sections are connected with swivel joints, enabling the chute to be easily moved horizontally as well as vertically across the full width or length of the site. The tower end of the chute is fixed at a height sufficient to make the descent of the concrete rapid; this may require the height to be anything up to 100 feet or more, depending upon the length of the chute and the quantity of water in the concrete; in no case should a fall of less than 1 in 4 be adopted. Intermediate supports for the chute consist of light A frames, or gin poles placed at the joints. Sometimes the whole length is suspended by ropes and pulleys attached to an overhead cable connected to the top of the hoisting tower at one end, and to a special tower or other structure at the other end. The concrete is discharged from the mixer into the bucket which is raised up the tower and automatically tripped and tipped into the chute hopper by means of a hoist operated by electricity, gasoline or steam power.

The popularity of chuting is entirely due to the great saving in time in delivering the concrete from the mixer to the forms. With a well operated plant the pouring is continuous and often between 50 and 60 cubic yards per hour are delivered with one plant.

When the chuting system was first boomed, in 1913-14, there was considerable opposition by certain architects

FIG. 31.—CHUTING CONCRETE.

Note the exceedingly long length of chute and imagine what the consistency of the concrete must be to enable it to freely flow the whole distance from the hopper to the trench.

and engineers owing to the fact that a large excess of water is required in the concrete to cause it to flow readily along the chute ; this opposition appears to have had very little effect, as since that time the system has become almost universal for extensive work. The rapidity of its use was chiefly due to the power of advertising ; the makers of chuting devices advertised them very extensively, making a strong point of their records of very low cost in the pouring of the concrete ; consequently, where time and economy of cost are the predominating factors, chuting is preferred.

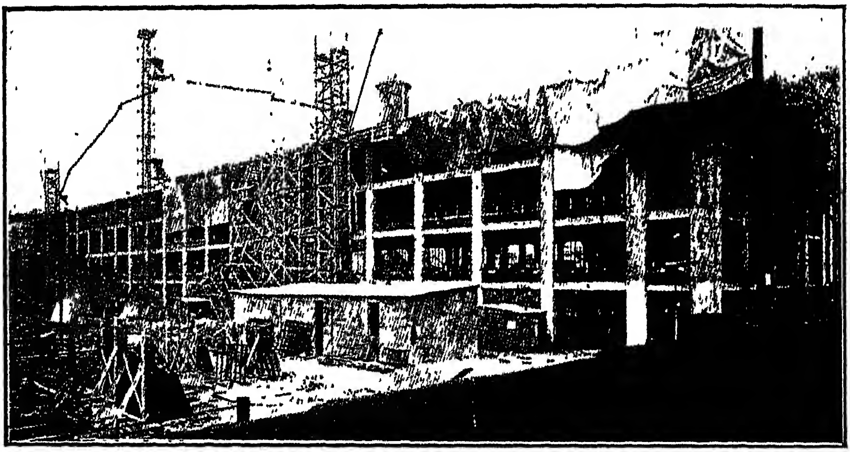


Fig. 32.—Chuting Concrete.
Showing the Chute suspended from overhead Cables.

In reinforced concrete work where the compressive strength of the concrete is relied upon to the limit, chuting should not be allowed ; the excess water causes porosity, reduces the strength, and retards the setting of the concrete ; it also affords an excuse to minimise or entirely dispense with the tamping. The longer the radius of the chute or the lower the tower the more water is required to assist the flow. The ideal quantity of water produces concrete that is sticky and pasty rather than free flowing. A mixture of this description cannot be chuted.

SEA-WATER CONCRETE.

Sea-water, or brackish water should not be used for reinforced concrete. There is no objection to its use for concrete not reinforced. From recent extensive examinations of various kinds of structures built of sea-water concrete in widely different parts of the world, including England, Scotland, Nova Scotia, Florida, California, South Africa, Greece, and Tasmania, it can be concluded that salt-water mixing has no deleterious effect on the

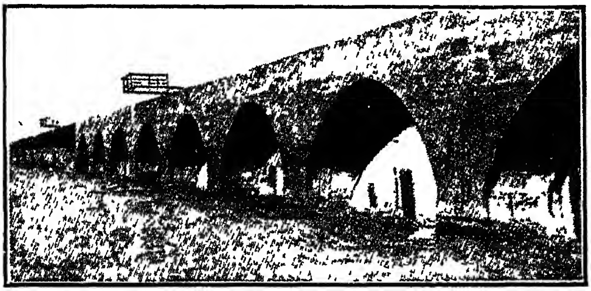


FIG. 33.—PIER ON THE EAST COAST RAILWAY, FLORIDA.
The concrete was gauged with sea water, and when recently inspected, after 15 years of service, was found to be in excellent condition

compressive strength of the concrete or upon the chemical action which takes place when cement sets, but in reinforced concrete work the presence of the salts may have an injurious effect upon the embedded steel in accelerating corrosion. Also if the structure is exposed to stray electric currents the salt will contribute largely to the disentegration from electrolysis. It is also found that if large masses are poured and the concrete is mixed very wet, a larger quantity of laitance will accumulate on the surface of the concrete than if fresh water were used. This added laitance consists chiefly of magnesium hydro-oxide precipitated from the sea-water.

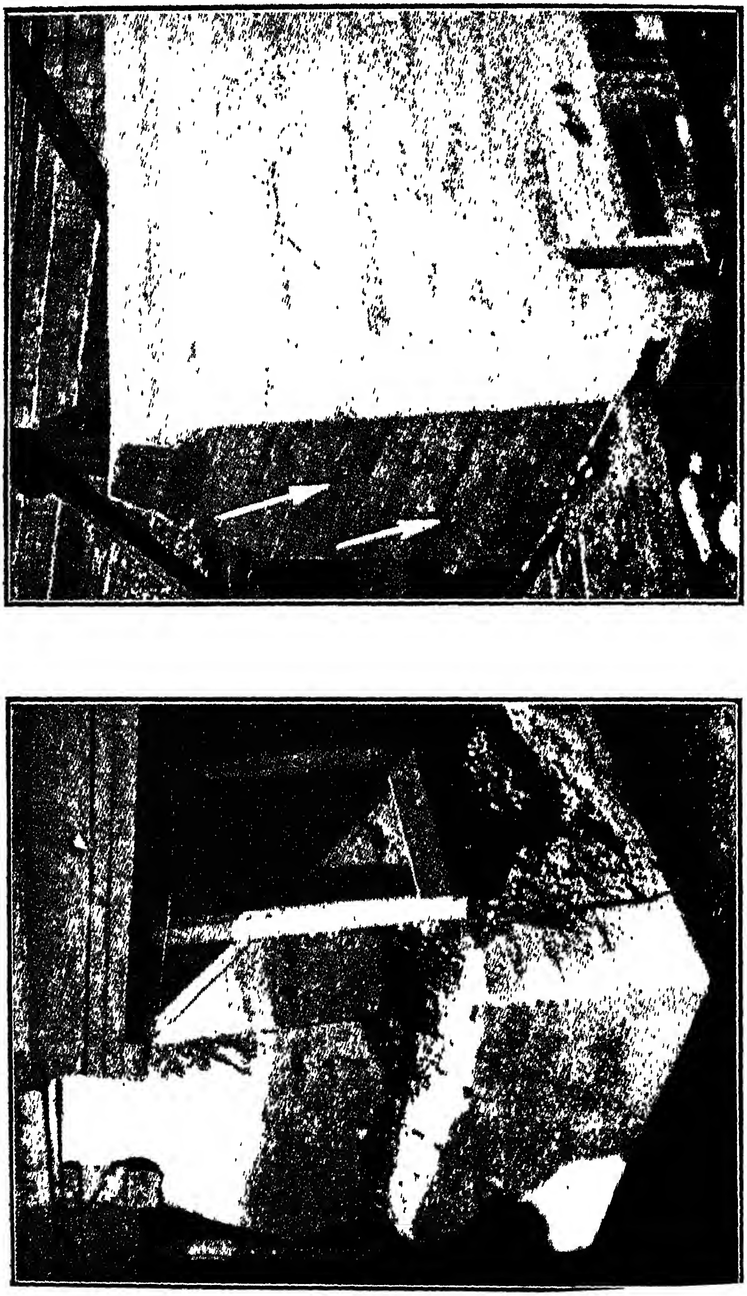


FIG. 34.
The right-hand view shows construction seams in a wall immediately after construction. The left-hand view shows how these seams were attacked by sea-water in 12 years.

The effect of mixing sea-water with concrete must not be confused with the effect of the penetration of sea-water. It is well-known that many concrete marine structures in all parts of the world are progressively deteriorating from chemical disintegration, which for many years has been receiving considerable attention from eminent engineers. A most extensive and complete study of this deterioration was recently made by Rudolph J. Wig, and Lewis R. Ferguson, of the United

FIG. 35.—THE FACE OF A SEA-WALL,
showing rapid disintegration caused by the surface being tooled off to give artistic effect.

States Bureau of Standards, and the Portland Cement Association. Their investigation covered a period of two years, and included the inspection and study of nearly every marine structure in the United States, and many in other countries. In addition to an examination of each structure, a study was made of the original specifications, drawings, records of construction and the materials used, and photographs made during construction, wherever these were obtainable. As a result of

this investigation the investigators stated that deterioration is taking place in concrete structures along all the coasts of the United States, Canada, Cuba, and European Countries, and that the action is far more rapid in cold climates than in the tropics, owing largely to the mechanical erosion from frosts. The disintegration proceeds at a very slow rate where there is no frost or severe mechanical erosion to remove the softened material.

Deterioration of Plain Concrete in Sea-water.—Messrs. Wig and Ferguson further stated that their investigation also showed that deterioration of plain concrete may be due to several causes, but the underlying one, which is

FIG. 36.—THE TOE OF A SEA-WALL,
showing rapid disintegration after the sharp corner was abraded.

chemical disintegration, cannot take place unless the original surface of the concrete is first abraided or eroded from mechanical means. The reason for this is that when the green concrete is placed in the forms, the lime of the cement at or near the surface is in a form to combine readily with carbon dioxide in the atmosphere; thus there is formed at the surface and for a slight depth in the mass, lime carbonate which is practically insoluble in water. This layer of insoluble material acts as an armour or protection to uncarbonated cement in the interior of the mass. If the concrete is deposited in water and not exposed to the air, there is sufficient

carbon dioxide in the water to carbonate the lime at the surface, and the same result is obtained. This explanation is supported by many physical, chemical and microscopic studies which have been made by various authorities.

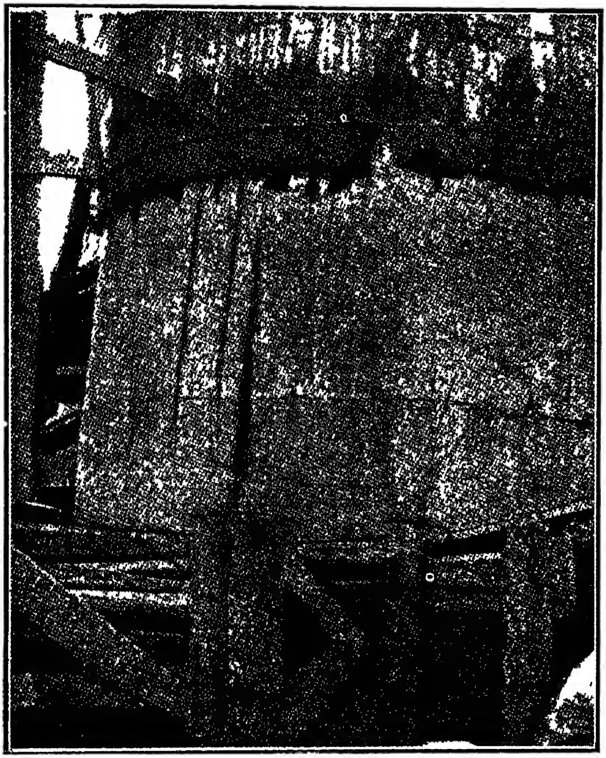


FIG. 37.—A PIER ON THE ATLANTIC COAST,
built in 1890 of concrete, made with White Bros.' English cement, being protected, the concrete, when recently inspected, was in excellent condition.

To secure the permanence of plain concrete in salt-water it is therefore necessary to protect it against mechanical abrasion. In designing the work sharp corners and edges should be avoided, and a form adopted that will offer the least possible opportunity for abrasion or erosion. Well made concrete thus protected is permanent in sea-water.

Reinforced Concrete in Sea-water.—The foregoing detailed investigation also showed that the deterioration of reinforced concrete in sea-water is undoubtedly due to the corrosion of the embedded steel above the water-line, and not to electrolysis which at one time was the most popular theory of this corrosion. The effect is more pronounced in some locations than in others. Certain conditions greatly accelerate the action. The temperature, exposed conditions, quality of concrete, and depth of embedded metal, all influence the rapidity of deterioration.

There probably have been instances where electrolytic action caused the corrosion of reinforcement in marine structures, but such cases are very rare and the effect of this action is different in one very marked particular from that which is occurring in practically all reinforced concrete that is deteriorating in sea-water. The cracking of concrete in sea-water always starts above the high-water line, although it may extend a little below this point as corrosion develops. Very seldom is it found to go down as far as the elevation of low water, and then only when the reinforcement is deeply embedded so that the spalling effect extends a considerable distance below where the metal is affected. In all experiments made to determine the action of electric current on reinforced concrete it has been found that cracking due to the corrosion of the steel starts where the current leaves the rods. Therefore, if electrolysis were the cause the corrosion and cracking must occur below high-water line, where the current leaves the metal. If the corrosion and deterioration do not extend below the water-line, electrolysis cannot be the cause.

The report further stated that the investigation undoubtedly showed that the real cause of the deterioration of reinforced concrete marine structures is the accumulation of salts in the pores of the concrete above the water-line by capillarity and evaporation, and the absorption by the concrete of sea-water laden air which penetrates to a far greater depth than was previously thought possible.

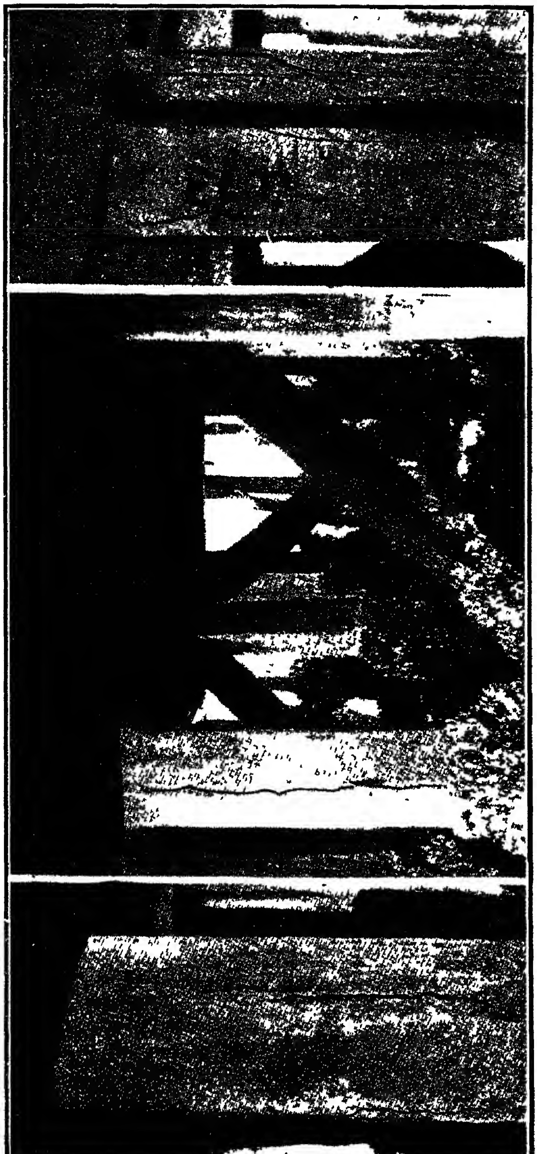


FIG. 38.

A few of 2 500 Precast Concrete Piles in a Pier on the Atlantic Coast; nearly every one was cracked or spalled through corrosion of the steel, when inspected 10 years after construction

Chlorine and oxygen together form a very active corroding agent for the steel, the great expansive force developed by this corrosion causes crackling and spalling of the concrete, and exposes it beneath the surface skin to the decomposing chemical action which rapidly takes place. In warm humid climates the corrosion of the reinforcement and the subsequent cracking of the concrete proceeds much more rapidly than in cool climates where the amount of moisture in the atmosphere is less.

To permanently secure reinforced concrete work in marine structures the following recommendations are given :—Every effort should be made to diminish the

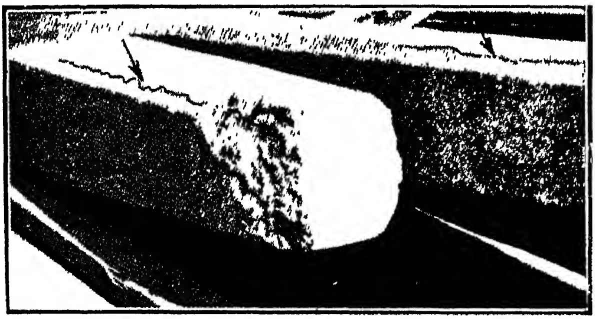


FIG. 39.

These reinforced concrete piles were made for a pier at Long Beach, California ; they were not used for the work, but were left on the beach high above the water, after a short time cracks appeared, caused by porous percolation and subsequent corrosion of the steel. Inspected by the Author

amount of reinforcement, especially above the water-line, and to use small rods rather than large ones, placing the rods not less than 2½ inches from the surface, and every possible precaution taken to get a good dense impermeable concrete, and to reject all materials, cement aggregate or water, which may tend to accelerate the sea-water disintegration. Only the very best aggregates should be used. All normal Portland cements if properly made and used will resist sea-water disintegration. There are numerous structures built many years ago, still in excellent condition, which support the foregoing statement.

All forms should be surfaced lumber and made tight to prevent leakage of the concrete, below water-line as well as above. In running water or where there is a wash, tongued and grooved lumber should be used. If the forms are oiled, which is preferable, mineral oil should be used. Use a richer mixture than for similar work on land. Use sufficient water to permit of light tamping, but not so dry as to require an effort to bring the water to the surface or to make it difficult to obtain compactness around the reinforcement. Take every precaution to prevent formation of construction seams of laitance. Protect the completed work from abrasion by wood fenders or other suitable means.

ALKALI ACTION ON CONCRETE.

During recent years engineers have been much concerned with the serious decomposition of concrete structures in the arid regions of America and in Western Canada. Foundations of many important buildings, numerous irrigation conduits, aqueducts, sewers and other underground structures, are being rapidly destroyed by the action of the alkali water in the subsoil. The author has inspected many of these structures, and has been responsible for the renewal or repair of a number of them. In some cases decomposing action commenced within a year after construction, portions of large trunk sewers have been so badly disentegrated as to collapse within four or five years after construction.

The salts usually found in these so called alkali waters are similar to those that are found in sea-water and are often found in considerable quantities in spring and river waters, They are chiefly sodium chloride, magnesium sulphate, calcium sulphate, sodium sulphate, and sodium carbonate.

During the last few years considerable money has been spent on investigations and experiments with a view to finding the actual cause of this decomposition and the most satisfactory treatment to prevent it. The conclusions of the various authorities are similar to those

given for the action of sea-water and include the following :—

(1) Portland cement concrete, if porous, can be disintegrated by the forces exerted by the crystalization of almost any salt in its pores; if a sufficient amount is allowed to accumulate, a rapid formation of crystals is caused by its drying. Porous stone, brick or other materials are disintegrated in the same manner. Therefore, a dense nonporous concrete is essential.

(2) Well-made concrete with an impervious skin when totally or partially immersed, is not subject to decomposition by alkaline water. The chemical action of various sulphate and chloride solutions is prevented by the carbonisation of the lime of the cement near the surface, or by the formation of an impervious skin or protective coating by saline deposits.

(3) There is no apparent relation between the chemical composition of a cement and the rapidity with which it reacts with alkaline or sea-water when in intimate contact.

(4) If the lime of the cement is carbonated it is practically insoluble. The quantity of alumina, iron, or silica present in the cement does not affect its solubility. The magnesia present is practically inert. The quantity of sulphur present up to 1·75 per cent. does not affect its solubility, but a variation in the quantity may affect its rate of hardening.

Methods to Prevent Decomposition.—Various authorities have suggested the use of different ingredients to be mixed with the concrete or to be applied to the finished surface with a view to their reacting with the salts or alkali and forming insoluble compounds, thus preventing the concrete from decomposition. Among the suggestions are :—An admixture of salts of barium to form, with soluble sulphates, insoluble barium sulphate. Both barium chloride and barium carbonate have been used by being finely ground and about 2 to 5 per cent. mixed with the cement. Also additions of various forms of iron oxides, calcium and magnesium hydrated lime, road oil, a mixture of silicate of soda

solution and fish oil, lime soap, alum, coal tar oil, and finely crushed marble with about 10 per cent. gypsum. These and many patented mixtures have been from time to time used, but so far the results have not been satisfactory. Some of the treatments appear to have delayed decomposition for a short period, but the author, after having made extensive experiments, enquiries and inspections, has concluded that there is not yet any mixture for this purpose that can be considered of permanent value. Concrete can be, and is made permanently secure from the decomposing action of salt or alkaline waters, but the only way at present known to make it so, is to use first-class materials and workmanship, aggregates of a siliceous nature, avoiding uncrystaline limestones, make use of every possible means to secure :—A dense impervious concrete, an absence of laitance seams and imperfect work joints, thorough protection of reinforcement, avoidance of frost action on green concrete, a richer mixture for, and perfectly smooth finish to, all exposed surfaces, and wherever possible, to apply on the surface a coat consisting of one part of cement to one part of sand finished to a glassy surface with a steel trowel.

EFFECT OF SEWAGE AND SEWAGE GASES.

Many large concrete sewers in America, Canada, England and Europe have been greatly damaged by decomposition of the concrete causing disintegration, in some cases so badly as to necessitate reconstruction after a few years of service. The author was responsible for the inspection, repair and part renewal of a combined sewage and storm water sewer in the province of Manitoba, Canada. The sewer was of egg shape section, varying in height from 2 feet 6 inches to 12 feet, and was at a depth below the surface varying from 5.feet to 20 feet. At the time of the inspection it had been built four years. The disintegration varied in different parts of the sewer, throughout its whole length of several miles, from patches of an inch or two up to large areas ; in

FIG. 40.

Fig. 41.

FIG. 42.

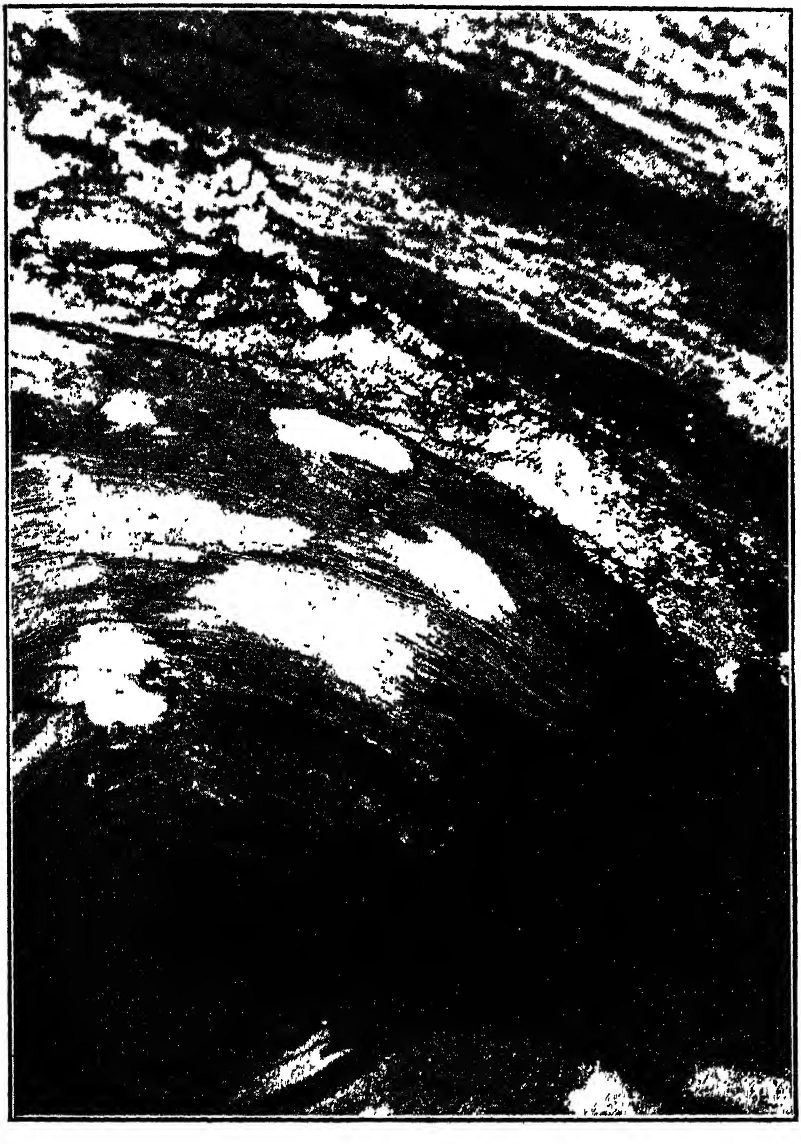


Fig. 43.

several places it had disintegrated so badly that the entire structure had collapsed. Many similar sewers in the same district had suffered as severely. Commencement of decomposition appears as a whitish crumbly crust on the surface ; this crust is easily rubbed off with the hands ; it develops into a moist light grayish mossy substance, and eventually into a pasty mass resembling slacked lime containing loose pieces of the original aggregate. The decomposition also varies with the nature of the stone.

The chief cause of this decomposition is found to be the excessive amount of sulphur in the sewer air, which is retained in the sewer for a lengthy period through lack of ventilation. The source of the sulphur is the hydrogen sulphide which is evolved naturally from sewage during decomposition. The sulphurous gas is oxidized by the air and so forms sulphuric acid, which immediately attacks the lime in the cement, and that in any uncrystalline limestone aggregate, converting it into soluble calcium sulphate. This sulphate, being of larger volume than the original lime, disrupts the concrete in forming, the action being similar to that of alkali crystals or repeated freezings. The resulting material gradually washes or crumbles away, leaving the hard aggregate to fall out.

In the Manitoba sewers the disintegration in some parts was due to sewer gas, and in other parts to the action of

Figs. 40, 41, 42 and 43 show various stages of the decomposition of a concrete sewer in Manitoba, Canada.

The concrete was made of a 1 to 6 wet mixture, and contained a considerable quantity of uncrystalline limestone ; it was very porous and insufficiently dense to resist the penetration of moisture and the deleterious sewage gases. The whole sewer was badly ventilated. The sewer air was found to contain a large proportion of carbonic acid and ammonia, and a small proportion of sulphurated hydrogen. The surrounding soil consisted of a plastic calcareous clay containing a small trace of alkaline matter, underlying about 18 inches of peaty vegetable earth in which there was found a trace of carbonic acid, probably generated by decaying vegetation and carried in by surface water.

Note the carbonate of lime stalactites in Fig. 40, and the cracks in the other figures. The white coating is carbonate of lime, powdery where dry and of a pasty consistency where wet. The rough patches are of a soft putty nature containing a few loose pieces of hard aggregate. In several places this sewer had entirely collapsed through weakness from entire disintegretion.

alkali from the surrounding soil, or the combined action of sewer gas and alkali. The concrete was very porous, and of an inferior material containing a considerable quantity of uncrystaline limestone. There was no means of ventilation other than the usual manholes, which were spaced at unusually long distances. The portion of the sewer sufficiently large to work in was repaired and the collapsed portions rebuilt. All decomposed patches were cut out and made good with new concrete ; the whole of the interior surface was thoroughly cleaned with a 15 per cent. solution of muriatic acid and water, then well washed with clear water ; after which all smooth portions were hacked to provide a key for a skin coat which was applied about $\frac{3}{8}$ inch thick, and consisted of one part Portland cement to $1\frac{1}{2}$ parts of clean fine sand. The surface being trowelled to a smooth glassy finish with a steel trowel. Better ventilation was also provided.

At the time of an inspection made by the author, six years after the repairs were finished, it was evident that the treatment was quite satisfactory, there being no signs in the interior of further disintegration of the portion so treated.

During the same period as that of the above examples, considerable concern was occasioned by the disintegration of a large semi-elliptical sewer in Chicago. This sewer was about 12 feet wide and about 16 feet high, with the concrete 12 inches thick at the sides and crown ; it had been built about seven years. The section where the destruction was particullary studied was a stretch of about 500 feet extending from the mouth of the sewer on the drainage canal. The concrete about the mouth was hard and dense ; a few feet inwards it began to show thin patches of a white powdery crust ; further in this crustation rapidly assumed the character of disintegration given way to a surface of loose gravel, held in a wet muddy matrix like lime putty, but apparently only above the water-line, that below being smooth and hard. An analysis, by the Kansas City Testing Laboratory, of the concrete taken from the top part of the sewer is given in the following table.

ANALYSES OF DECOMPOSED CONCRETE IN A CHICAGO SEWER.

	Sound Concrete Percentage.	Decomposing Concrete Percentage.
Moisture	2 13	4 12
Loss on ignition	19 18	20·21
Silica (SiO_2)	36 38	30·75
Iron (Fe_2O_3)	3·53	1·32
Alumina (Al_2O_3)	9·04	6 40
Lime (CaO)	23 36	17 30
Magnesia (MgO)	3 38	3 49
Sulphur (SO_3)	2·37	16 62
	100 00	100 00
Water soluble	5 12	28 96

Sulphur in 200 mesh material from decomposing concrete 46·30%
Sulphur in ¼-inch material from decomposing concrete 11 44%

A comparison shows that the concrete had absorbed about six times its original content of sulphur, or probably more, as the sound concrete tested was not entirely free from the whitish crust. The moisture and organic matter had naturally increased ; the other items had naturally decreased proportionately owing to the large increase in sulphur. Some of the lime which had changed into soluble sulphate had evidently washed away. The decomposition was thought by some to be due to alkali in the soil surrounding the sewer leaching through the concrete. The soil was found to contain about 0·1 per cent. sulphates of magnesium and calcium, which amount was considered to be enough to affect the concrete if concentrated by evaporation. This theory, however, could not be accepted, as it would not account for the circumstances that in the part of the sewer under street pavements properly drained by the usual catch-basins and vetrified pipe to the sewer, and where the opportunity for evaporation is small, the disintegration

was much further advanced than in that part of the sewer under the lawns and other locations where the rain could more easily seep through to the concrete, and where fresh air has more access to the interior of the sewer for evaporation, also for the fact that the sewer below the water-line had remained in good condition. A sample of the atmosphere taken from just under the sewer crown and about 150 feet from the mouth, showed 2·45 mg. of sulphur per litre of air, equivalent to 0·17 per cent. by volume in the form of sulphur dioxide or hydrogen sulphide, which quantity is found to be sufficient to be very active chemically. The sewage carried was most unusual in character, it included more than half of the entire waste from the packing-house district of the Chicago stockyards, and was very highly septic. The sewer formed an outlet from a creek to the main channel of a branch of the river ; the sewage before reaching the sewer was detained in this creek for a period of probably 36 to 48 hours ; it was therefore considered that the sewage on reaching the sewer must often have been very full of hydrogen sulphide. No special means of ventilation had been provided. The investigation of this case resulted in the same conclusions as was drawn in the Manitoba and other cases of disintegration of sewers, which include the following :—With a sulphur content sufficiently high the sewage will throw off gases which, when mixed with the oxygen of the air and in the presence of moisture, form an acid sufficiently strong to attack concrete that is in any way of inferior quality, either in workmanship or materials. To resist the decomposing action the concrete must be of first quality, dense and non-porous throughout, with a smooth hard and impervious surface. Good ventilation must be provided so that there will be a good current of fresh air always drawing through the sewer ; this is particularly necessary where the sewage is made up largely of trade wastes. Numerous sewers in America, Canada, England and Europe have been made of such concrete, and after being in use many years show no signs whatever of decomposition.

ADVANTAGES OF CONCRETE FOR SEWERS.

Particular advantages of monolithic concrete over brick for sewer construction are :—It is lower in cost, about 10 to 15 per cent. ; it may be moulded to any form desired ; the thickness can be varied as often as desired to suit pressures and local conditions ; the natural earth under the sewer is not disturbed and as the concrete fills all irregularities in the soil a better foundation is obtained ; the structure can be made to develop strength when subject to strains not predetermined ; it can be made impervious by using carefully selected materials and care in mixing and placing, and a smoother surface is obtained through the illimination of joints resulting in a higher capacity.

CEMENT.

The quality of the Portland cement is of the utmost importance. Architects and engineers generally specify it to at least comply with the requirements of the standard specification of the leading national engineering institute or material testing society. In America it is known as the specification of the American Society for Testing Materials ; in England, the British Standard Specification ; in Canada, the Specification of the Engineering Institute of Canada.

No particular brand should be accepted without being subjected to a number of careful laboratory tests for fineness, time of setting, expansion, compression and tension. Some specifications also call for specific gravity, and sulphuric acid and magnesia tests. The expansion and compression results are the most important, but owing to the greater difficulty in carrying out compression tests than tension tests it is most usual to accept on the tension tests. It is, however, found that the tensional resistance of a cement bears a certain relation to its compressional resistance ; therefore a

cement with a satisfactory compressional resistance can be selected ; then the tensional resistance of the same cement determined, and this specified for the tension acceptance tests of each delivery.

A good quality of cement for reinforced concrete work should at least comply with the following conditions :—

(1) **Fineness.**—A residue of not more than 8 per cent. by weight shall be retained on a 100 mesh sieve, and not more than 22 per cent. on a 200 mesh sieve.

(2) **Time of Setting.**—It shall not develop initial set in less than 45 minutes when the Vicat needle is used, or 60 minutes when the Gilmore needle is used. Final set shall not be attained in less than 3 hours nor more than 10 hours.

(3) **Expansion.**—This is sometimes called "Constancy of Volume." The description of this will depend upon the method of testing. A common and convenient method is that known as the pat test, which may be described as follows :—Pats of neat cement, about three inches in diameter, half an inch thick in the center, tapering to a thin edge, shall be kept in moist air for a period of 24 hours, then subjected to the following tests.

(a) A pat shall be kept in air at a normal temperature, and observed at intervals during a period of at least six days.

(b) A second pat shall be kept in water maintained at a temperature, of as near as possible, 70 degrees F., and observed at intervals during at least six days.

(c) A third pat shall be exposed in an atmosphere of steam above boiling water, in a loosely closed vessel, for five hours.

To be satisfactory these pats should be firm and hard, and show no signs of shrinkage, cracking, distortion or disintegration, at the end of the testing period.

(4) **Compression.**—One day kept moist, and six days in water, neat cement to be 3,700 lb. per square inch, for three parts standard Ottawa sand and one part cement, 2,500 lb. per square inch.

One day kept moist, twenty-seven days in water, neat

cement to average 5,200 lb. per square inch, or 10 per cent. increase after the seven days ; that of three parts sand and one part cement to average 3,000 lb. per square inch.

(5) **Tension.**—One day kept moist and six days in water, neat cement to average 500 lb. per square inch ; for three parts of Ottawa sand and one part cement, 200 lb. per square inch.

One day kept moist and twenty seven days in water, 600 lb. per square inch, that of three parts sand and one part cement to average 300 lb. per square inch.

In some specifications the tensile strength for neat cement is now omitted ; as the cement is not used neat in actual work, this test is considered to be of less value than the sand test. Some engineers specify the tests to be made with the actual sand that is to be used on the works. The allowable stresses for designing are determined accordingly.

(6) **Specific Gravity.**—The specific gravity of the cement should be not less than 3·10 ; should the test fall below this requirement it is sometimes specified that a second test may be made on a sample ignited at a low red heat ; the loss in weight of this ignited cement is not to exceed four per cent.

(7) **Sulphuric Acid and Magnesia.**—The cement shall not contain more than two per cent. of anhydrous sulphuric acid ($S0^3$), nor more than five per cent. of magnesia ($Mg0$).

Every consignment should be tested, and if not up to the acceptance standard it should not be allowed on the job.

A good lock-up shed should be provided on the job for storing the cement, and only such quantities should be stored as can be used within two weeks after the time of testing. On no account should the cement be exposed to the air by spreading it out on a floor or in any other way. If it has passed the specified tests it is fit for immediate use, and further exposure, especially with the unfavourable conditions existing on building works, is petrimental to its strength.

REINFORCEMENT.

Care must be exercised in placing the bars, which must be in exact accordance with the specifications and drawings, as a slight error, either in the size, distance apart, or depth below the surface of the concrete, may lead to serious results.

The metal used should be mild steel, with an ultimate strength of not less than 60,000 lb. per square inch, and the yield point between 50 and 60 per cent. of the ultimate strength. Steel is, however, rolled with an ultimate tensile strength up to 105,000 lb. per square inch ; consequently, with a factor of safety of four on the ultimate strength, the working stress may be any value between 15,000 lb. and 26,250 lb. per square inch, according to quality. The usual allowance is 16,000 lb., or at most 17,000 lb. per square inch.

There is no advantage in using a high carbon steel or one that will develop a higher tensile resistance than 70,000 lb. per square inch, unless the yield point is correspondingly high, and this cannot be relied upon with steel above this strength.

The working stress for both steel and concrete should be estimated from their yield points, for these are really the critical points, if it is reached in one before it is reached in the other, fracture of the concrete will occur through the adhesion of the concrete to the steel being destroyed ; therefore, the working stress should be governed by these limits ; it is, however, usual to estimate from the ultimate resistances, which is done in this work to conform with ordinary practice, but the yield point should be specified to be at least 50 per cent. of its ultimate tensile resistance.

No welding should be allowed, for in this there is danger of the sectional area being diminished, and of imperfect welds that are often most difficult to detect.

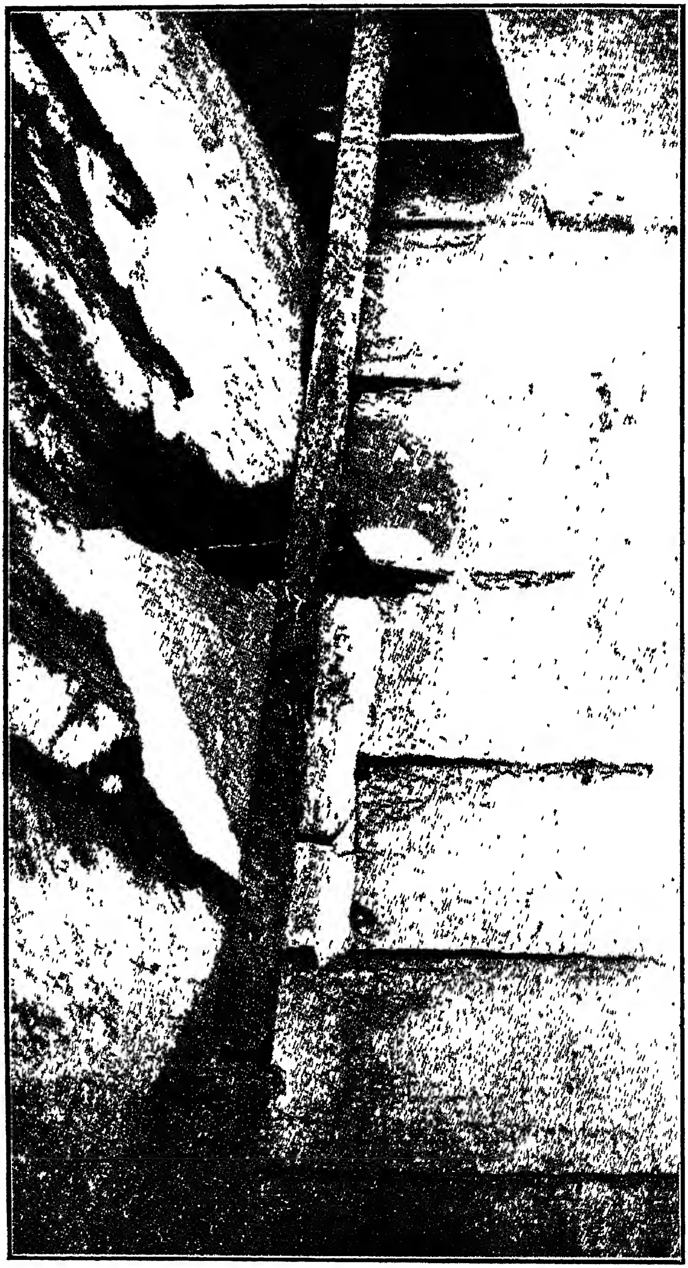


Fig. 44.—Result of Insufficient Cover to the Bars.

The partition wall and ceiling of a water tank showing cracking and spalling of the concrete, due to corrosion of the steel, after 12 years of service. In places the surface of the bars were not more than $\frac{1}{2}$-inch from the face of the concrete. No defects appeared below the water line.

Deformed Bars.—There are many varieties of deformed steel bars manufactured, especially for reinforced concrete work. The object of these is to supplement the adhesion of the concrete to the steel by a mechanical bond ; due to the shape of the bar, the computations, however, in this work, are based on plain bars. See information under " Adhesive Stress."

ADDITIONAL INFORMATION AND DATA FOR DESIGNING.

For warehouses, factories, or large buildings, the floor slabs should be not less than five inches thick, including not less then $\frac{3}{4}$ inch of concrete under the reinforcement. For lighter buildings the floor slabs may be as thin as four inches, including not less than $\frac{1}{2}$ inch of concrete under the reinforcement. Beam reinforcements should have a bottom cover not less than 1 inch, or less than the diameter of the rods ; in no case is more than 2 inches necessary for beams under ordinary conditions ; at the sides the cover must be not less than 1 inch, and need not exceed $1\frac{1}{2}$ inch. For columns, the cover must not be less than $1\frac{1}{2}$ inch, or less than the diameter of the vertical rods, if these should be more than $1\frac{1}{2}$ inch. In footings, the steel should have not less than a 2-inch cover. All angles of the concrete should be splayed or rounded, as shown by the forms, Figs. 20 and 21, for sharp angles are easily injured, and spall off under heat.

The diameter, or least thickness of longitudinal rods in beams, and rods in slabs, should not be less than $\frac{1}{4}$ inch ; for any other reinforcement in beams the least diameter, or least thickness, must not be less than $\frac{1}{8}$ inch, and in slabs 1/10 inch. Reinforcement in the form of mesh must be sufficiently open for the coarse material to pass through. Compressive reinforcement should be anchored by stirrups, or otherwise, to $\frac{2}{3}$ the depth of the beam ; the anchors should not be further apart than 24 diâmeters of the reinforceing rods. Rods must not be further apart than 12 inches, or nearer together than double the diameter of the rods, in no case nearer than one inch. Rods which cross or lap should be wired together with soft wire. For length of lap see page 193.

Small rods fairly close together are better than large rods far apart, especially so in fire-resisting structures. In floor slabs, and similar work, rods about four inches apart are better than rods of double the sectional area placed 8 inches apart ; the concrete affords better protection, expansion and contraction of the materials will be minimized, and more uniformly distributed, and the adhesion area of the steel will be greater, thus affording greater resistance to the shearing action that takes place around the bars.

The surface of the steel should be clean and free from scale, and must not be galvanized, or coated with oil paint, tar or oil, but Portland cement wash may be used, which will assist in preventing oxidation, and will improve the adhesion between the concrete and steel.

WATERPROOFING.

If the structure is to be exposed to water, or to soil permanently wet, special precautions must be taken to prevent the moisture penetrating sufficiently to come into contact with the steel, for the reason explained on page 70. There are many so-called waterproof compounds manufactured for this purpose, some being applied as a surface wash ; these are nothing more than temporary preventatives ; others are mixed with the concrete, some of which appear to be fairly satisfactory, but they have not been in use long enough for their durability to be known.

According to a report of a committee of the American Society of Testing Materials, after extensive tests and laboratory experiments with properly selected and graded aggregates in mixtures ranging from 1 cement, 2 sand, and 4 stone, to 1 cement, 3 sand, and 6 stone ; a watertight concrete can invariably be produced. Even with sand of poor granulometric composition in mixtures of 1 cement, 2 sand, and 4 stone, permeable concrete is seldom found, and is very rarely, if ever, found with a 1-2-6 mixture. But in spite of this it is a fact that in

actual construction permeable concrete is quite common, even with a 1-2-4 mixture. This permeability the committee attributed to the following :—

(1) Defective workmanship, resulting from improper proportioning, lack of thorough mixing, separation of the coarse aggregate from the fine aggregate and cement in transporting and placing the mixed concrete, lack of density through insufficient tamping or spading, and improper bonding of the work joints, etc.

(2) The use of imperfectly sized and graded aggregates.

(3) The use of excessive water, causing shrinkage cracks and formation of laitance seams.

(4) The lack of proper provision to take care of expansion and contraction, causing subsequent cracking.

INTEGRAL TREATMENT.

The foregoing mentioned committee further investigated a sufficient number of the special water-proofing compounds on the market, as well as the use of certain finely divided mineral products, such as finely ground sand, colloidal clays, hydrated lime, etc., for the purpose of forming a general idea of the value of the different types ; the report on these was as follows :—

(1) The majority of patented and proprietory compounds tested have little or no immediate, or permanent, effect on the pemeability of concrete, and that some of these even have an injurious effect on the strength of mortar and concrete in which they are incorporated.

(2) In view of their possible effect, not only upon the early strength, but upon the durability of concrete after considerable periods, no integral water-proofing material should be used unless it has been subjected to long-time practical tests, under proper observation, to demonstrate its value, and unless its ingredients and the proportion in which they are present are known.

(3) The permanent effect of such integral waterproofing additions, if dependent on the action of organic compounds, is very doubtful.

(4) In general, more desirable results are obtained from

inert compounds acting mechanically than from active chemical compounds whose efficiency depends on change of form through chemical action after addition to the concrete.

(5) Void-filling substances are more to be relied upon than those whose value depends on repellent action.

(6) Assuming average quality as to size of aggregates and reasonably good workmanship in the mixing and placing of the concrete, the addition of from 10 to 20 per cent. of very finely divided void-filling mineral substances may be expected to result in the production of concrete which under ordinary circumstances of exposure will be found impermeable, provided the work joints are properly bonded, and cracks do not develop on drying, or through change in volume due to atmospheric changes, or by settlement.

External Treatment.—It is found that in large work, no matter how carefully the concrete has been made, cracks are apt to develop, due to shrinkage in drying out, expansion and contraction under change of temperature and moisture content, and through settlement. It is therefore often advisable on important work to anticipate and provide for the possible occurrence of such cracks by external treatment with a protective coating. Such coating must be sufficiently elastic and cohesive to prevent the cracks extending through the coating itself. The application of merely penetrative void-filling liquid washes will not prevent the passage of water due to cracking of the concrete. The committee, therefore, divided surface treatment into two heads :

(1) Penetrative void-filling liquid washes.

(2) Protective coatings, including all surface applications intended to prevent water from coming into contact with the concrete.

While some penetrative washes may be efficient in rendering concrete water-proof for limited periods, their efficiency may decrease with time, and it may be necessary to repeat such treatment. The committee expressed its belief that the first effort should be made to secure a concrete that is impermeable in itself, and that pene-

trative void-filling washes should only be resorted to as a corrective measure.

While protective extraneous bituminous or asphalt coatings are unnecessary, so far as the major portion of the concrete surface is concerned, provided the concrete is impermeable, they are valuable as a protection where cracks develop in the structure. The committee therefore recommended that combination of inert void-filling substances and extraneous water-proofing be adopted in especially difficult or important work.

Bituminous or Asphaltic Coatings.—Considering the use of bituminous or asphaltic coatings, the committee found that :—

(a) Such protective coatings are subject to more or less deterioration with time, and may be attacked by injurious vapours, or deleterious substances in solution in the water, coming in contact with them.

(b) The most effective method for applying such protection is the setting of a course of impervious brick, dipped in bituminous material, into a solid bed of bituminous material or the application of a sufficient number of layers of satisfactory membraneous material cemented together with hot bitumen.

(c) Their durability and efficiency are very largely dependent on the care with which they are applied. Such care refers particularly to proper cleaning and preparation of the concrete to ensure as dry a surface as possible before application of the protective covering, care should also be taken in the lapping of all joints of the membranous layers, and their coating with the protective material.

So far the committee had considered only concretes of the unusual proportions, viz., those ranging from 1 cement, 2 sand, and 4 stone, to 1 cement, 3 sand and 6 stone. It was suggested that impermeable concretes could be assured by using mixtures richer in cement. It was also suggested that the presence in the cement of a larger percentage of very fine flour might result in the production of a denser and more impermeable concrete, through the formation of a larger amount of colloidal cells.

In conclusion the committee pointed out that no addition of water-proofing compounds or substances could be relied upon to completely counteract the effect of bad workmanship, and that the production of impermeable concrete could only be hoped for where there is determined insistence at all times on good workmanship.

THE AUTHOR'S CONCLUSIONS.

As a result of many years' experience, extensive experiments, tests and observations of existing structures, the author has concluded that the most satisfactory results are obtained by using a high-grade cement ground extremely fine, a plastic consistency without an excess of water, the face of the work richer in cement, sand consisting of irregular size grains and in quantity a trifle more than required to fill all the voids in the coarse material, the whole aggregate mixed and deposited as explained on page 30. Small rods should be used and placed not nearer the surface than 1½ inches, and wired to distribution bars placed not further apart than 18 inches, and after the forms are removed, render, if possible, the surface with a ¼ inch coat of fine stuff, consisting of one part of cement to one part of clean fine sand, to which is added 5 per cent. of hydrated lime to the weight of cement run and passed through a fine sieve ; finish by trowelling off with a steel float to a smooth glassy surface. Work of this description will permanently resist the penetration of water to the steel under a very considerable pressure.

Hydrated lime is a flucculent powder produced by slacking quicklime under standardized factory conditions ; after being wetted it loses its granular character and is converted into a smooth paste ; its use increases the mobility of the concrete, reduces the possibility of visible physical defects, acts as a void-filler, will cause the concrete to slide down the chute at a smaller angle, and the concrete will fill the forms more perfectly and will surround the reinforcement more closely. It slightly retards the drying of the concrete which is often an advantage.

Apart from the above method the only satisfactory treatment is to apply on the surface, if permissible, if not, and if practicable, within the thickness of the concrete, ¾ inch of good asphalt sufficiently elastic to resist without cracking the expansion and contraction to which it will be subjected.

ESTIMATING LOADS.

In estimating the load to be carried by any structure, the weight of the structure itself must be taken into account together with the superimposed load, and an allowance that may be considered necessary to provide for vibration or shock, likely to be caused by moving machinery, traffic or wind.

The weight of reinforced concrete may be taken as follows :—

Gravel or stone	150 lb. per cubic foot
Slag or brick	140 lb. ,,
Cinder or coke-breeze	130 lb. ,,

Floors should be designed for the following loads, which include sufficient to take care of vibration, but to which must be added the weight of the floor itself.

	lb per sq. ft.
Dwelling houses	50
Asylum and hospital wards, lodging-house and hotel bedrooms, schools, stairways, flats and apartments	75
Offices, banks and hotels	100
Churches, concert lecture and reading rooms, retail shops and general stores, workshops, stables and coach-houses, foot-bridges	115
Public assembly halls, music halls, theatres and corridors of public buildings	125
Museums, ball-rooms and drill-halls	150
Factories, with special strengthening where required for machinery, libraries, book-stores and warehouses	200
Wind on vertical surfaces in towns or fairly sheltered positions	30
Wind on vertical surfaces in exposed positions	45
Roofs of less than 20 degrees pitch, exclusive of weight of structure	30

Roofs over 20 degrees pitch, weight of structure plus an allowance for snow and a horizontal wind pressure of 45 lb. per square foot.

For buildings of more than two stories, exclusive of warehouses and such buildings as are likely to have all

their floors loaded at the same time, the loads for the floors, columns, walls, etc., at different levels may be estimated as follows :—For the roof and topmost storey, the usual full load to be taken ; for the second storey below the top, 10 per cent. less than the usual full load ; for the third storey below the top, 20 per cent. less ; and so on by increments of 10 per cent. per storey until the reduction amounts to 50 per cent. of the usual load, after which no further reduction shall be made, but 50 per cent. taken for the remaining stories.

Although the above specified loads and reductions are safe amounts to use in designing, it is advisable for the designer to consult the building ordinance of the district where the building is to be erected, as most of these specify the loads and the permissible reductions for that particular district.

Test Loads.—No weight approaching the working load should be allowed on a structure, after the supports have been removed, until at least four weeks from the time the concrete was deposited in place. No test load should be applied until at least eight weeks from the time the concrete was deposited in place, and it should then not exceed $1\frac{1}{2}$ times the load for which the structure was designed, or $1\frac{3}{4}$ times the load at the end of four months. At no time must a test load be sufficient to cause the stress in the steel to exceed threequarters of its yield point value. The deflection, while supporting the test load, should not exceed 1/600th of its span.

The concrete is not to be used in tension, although when perfect it is capable of resisting about 200 lb. per square inch, it cannot be depended upon, as a slight crack will entirely destroy this resistance. Apart from this, structures are generally designed to resist from 80 to 100 times as much tension as the concrete will take ; consequently, when the structure is carrying its full load the concrete in the tension area is comparatively useless.

DATA FOR DESIGNING.

In order to design a beam, or to determine the strength of an existing beam, the following data are required :—

(a) Working stresses of the materials, per square inch, which may be taken as follows, the values being considered as one-fourth of the average ultimate resistances, the concrete being a 1-2-4 gauge :—

	Compression	Shear	Adhesion to Steel
Stone, gravel and selected slag concrete	600 lb.	60 lb.	100 lb.
Brick, average limestone, and ordinary slag concrete ...	500 lb.	50 lb.	80 lb.
Coke - breeze and cinder concrete . .	250 lb.	25 lb.	35 lb.
	Tension	Shear	
Steel . . .	16 000 lb.	12 000 lb.	

(b) The ratio of the moduli of the elasticity of steel and concrete, which equals the modulus for the steel divided by the modulus for the concrete, equals $\frac{Es}{Ec}=m$. The following moduli are in lbs. :—

Stone, gravel, and selected slag $m = \frac{30\ 000\ 000}{2\ 000\ 000} = 15.$

Brick, average limestone or slag $m = \frac{30\ 000\ 000}{1\ 660\ 000} = 18.$

Coke breeze and cinder $m = \frac{30\ 000\ 000}{1\ 000\ 000} = 30.$

Further information for designing is given under "Principles of Design," and with various examples throughout this work.

PRINCIPLES OF DESIGN.

For an explanation of the symbols used throughout this work, refer to list at end of book.

In designing the chief consideration is strength ; this, however, should always be considered together with economy. Economy in design has sometimes to be considered from two points of view, the engineering and the architectural, which points will not always coincide. The most economical engineering structure would have

a certain arrangement of beams, slabs and columns, all spaced and proportioned in the most efficient manner, and with a definite percentage of reinforcement, all determined with due regard to the loading, and with a view to obtaining the strongest, most satisfactory and cheapest structure. But if these engineering points were the only ones considered by the architect, the result in some cases would be very uneconomical. The engineering structure may be considered either a plain mass or a skeleton framework devoid of architectural embellishments. The architectural structure is the engineering structure made more presentable by the addition of fittings and embellishments. Economy on the engineering side is purely structural and, in a building, is attained by keeping the floor slabs thin, by a free use of beams or columns, and by keeping the beams thin and deep, also by graduating the columns in size according to their different loads. This from an architectural point of view might turn out very uneconomical as it might involve so much additional finishing in the way of cornices and other details, also extra expense to secure sufficient light and ventilation than would be the case if thicker slabs with fewer and more shallow beams were used. Therefore, if we wish to design economical structures there are many factors to be considered, the most important of which are those that influence all structures and should be considered by both the architect and engineer. They are as follows :—

(1) The ratio of breadth to depth of beams.
(2) The percentage of reinforcement.
(3) The general arrangement, or lay-out, of beams and columns.

With regard to the first factor, the most economical section for rectangular beams is when the breadth is one-third of the depth ; but this gives a rather deeper beam than is desirable for most purposes. To have less depth means an increase of width, which would be placing the concrete in a less effective position ; consequently, a larger section would be required ; the increase in volume

and cost, however, is very slight until the width exceeds six-tenths of the depth, which is a good proportion for general purposes, and is about the proportion largely adopted.

Beams with double reinforcement are seldom as economical as beams with single reinforcement, whether they are so or not depends upon the ratio of breadth to depth, the percentage of steel, and the ratio of top reinforcement to bottom reinforcement. The depth of a beam with double reinforcement is less than the economic depth, or less than would be required if a beam with single reinforcement were used. Top reinforcement is added to assist the concrete in taking the compression, owing to the section of the beam, for some reason, having to be kept down, thus providing insufficient concrete to take all the compression. But compressive reinforcement is always very lightly stressed, seldom to more than 7,500 lb. per square inch ; consequently, a comparatively large proportion of steel is required to make good for a small decrease in the beam depth.

Regarding the second factor, i.e., the percentage of reinforcement, attention to which is of the utmost importance for great waste is often occasioned through an excess of steel being used. There is a certain percentage of steel to concrete that will give the most economic section ; it is the amount that will allow both the steel and concrete to be stressed to their allowable limits at the same time ; for instance, if the allowable unit stress in the steel is 16,000 lb., and that for the concrete 600 lb., which are the usual values, the concrete and steel should be so proportioned as to allow these stresses to exist when the structure is fully loaded ; we then get the full value out of each material. If one of the materials is understressed it means there is an excess of that material, and consequently a waste. Now for different classes of concretes there will be different percentages of steel required to give the desired result. The difference will depend on the ratio of the moduli of elasticity of the various concretes and the steel. For hard stone, granite, or gravel concrete, this ratio is 15, for which the economic

percentage of steel is, for the above allowable stresses, 0·675. For brick, or slag concrete the ratio is 18, and the steel percentage is 0·585. For cinder concrete the ratio is 30, and the steel percentage 0·25. See further information under Beam Design. From this it follows that in a beam, retaining wall, or any part of a structure under a bending stress, built of hard stone concrete, if single reinforcement is used, the sectional area of the steel should be 0·675 per cent. of the sectional area of the concrete. If there is more than this it cannot be fully stressed unless the concrete is overstressed, the excess is therefore waste. If there is less than 0·675 per cent. the concrete cannot be fully stressed unless the steel is overstressed. The reason of this is as follows :—In any structure under a bending stress the tension and compression are equal, and the tension at any distance from the neutral axis is equal to the compression at the same distance the other side of the axis ; also, the stress in the steel at any point is equal to the stress in the concrete at the same point, or at the same distance from the neutral axis of the beam, multiplied by the ratio of elasticity. Therefore, if the ratio is 15, the stress in the steel is 15 times as much as the stress in the concrete immediately surrounding it, and it cannot be stressed more under any consideration unless the stress in the concrete is increased. This is why compressive reinforcement in doubly reinforced beams is always so much understressed.

The elastic properties of the materials and how the stresses are affected thereby, is a very important consideration, and should be thoroughly understood before proceeding with the study of designing.

As a simple example in further consideration of this elasticity problem, refer to Fig. 45, which represents a block of concrete with steel rods embedded therein, similar to a column, and the whole perfectly level on the top. If a pressure be gradually applied, evenly distributed over the concrete and steel, and the elasticity of the steel is 15 times that of the concrete, there will be 15 lb. on every square inch of steel when there is ·1 lb.

pressure on every square inch of concrete ; this is b
it takes 15 times as much to overcome the elasticity
bring full effect on the steel as it does to bring full effect on the concrete ; consequently, the weight in pounds required to stress the whole of the concrete to 1 lb. per square inch, will be 1 lb. for every square inch of concrete, plus 15 lb. for every square inch of steel.

From the foregoing statement it is evident that if the axis is at the half-depth, so that the steel is the same distance below the axis as the extreme edge of the concrete is above it, the stress in the steel will be 15 times as

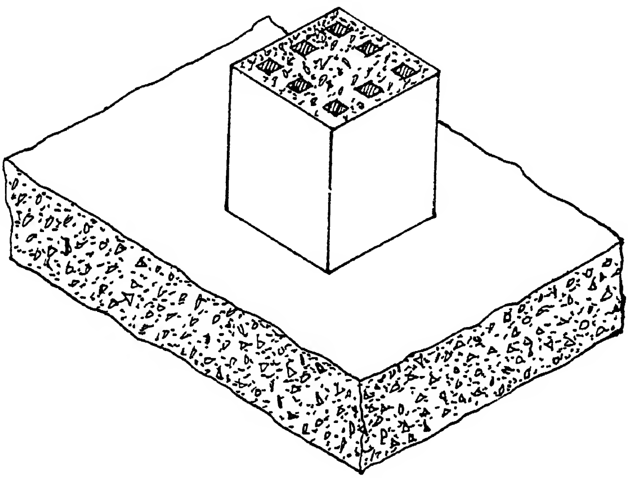


Fig. 45.

much as the stress in the top of the concrete ; which is little more than half its limit ; therefore, as the steel takes all the tension, much more steel is required than if it could be higher stressed, but to be so it must be further from the axis ; the exact distance will depend upon the ratio of elasticity, and the allowable unit stresses. The higher the steel is stressed the less steel will there be required to take the whole of the stress with the same quantity of concrete, and the further will the steel be from the axis. From this it follows that for the steel and concrete to be fully stressed to their respective limits

percentag utral axis must be somewhere above the half-0·675. n, or further from the steel than from the compression the urface, to which it approaches as the stress in the steel r increases. Therefore, the position of the axis varies according to the value of the ratio of elasticity and to the proportion of steel to concrete. From this we see the error of the early designers of reinforced concrete in assuming the axis to be at the half-depth, we also see that for the steel to be stressed to a given amount it must be at a definite distance from the axis, and of a certain sectional area, also that a definite area of steel is required to enable definite stresses to be developed in both the concrete and steel.

Knowing the foregoing fundamentals, there is not much difficulty in designing a beam. We know the factors which govern the position of the neutral axis, we also know how the stresses vary with the depth, and that all the compression is taken by the concrete above the axis, and all the tension by the steel below the axis. With this knowledge it is only a simple mathematical problem to design a formula whereby we can determine the section of a beam that will contain sufficient concrete above the axis to take the compression ; after which we have only to add the known percentage of steel.

In doubly reinforced beams the proportion of steel to concrete increases as the ratio of the compression bars to the tension bars increases. When the compression and tension bars are equal, and the allowable stresses are 600 lb. for the concrete, and 16,000 lb. for the steel, the economic percentage, for either compression or tension bars, is 1·08, or 2·16 for the whole reinforcement. This class of beam can be designed in a similar manner to beams with single reinforcement ; for which see examples of beams with double reinforcement.

BEAM DESIGN.

With all beams the moment of resistance at any point must equal the bending moment at the same point, and the total compression will equal the total tension. Referring to Fig. 46, the compression area is all that

portion above the neutral axis, it equals $b\ n$. The whole of this area is stressed, but the intensity is greatest at the top, and diminishes as the depth diminishes, to nil at the axis, as shown by the shaded portion ; the total compression can therefore be compared to this shaded portion. Hence, if c equals the compression at the top, the total compression will equal $\frac{1}{2}$ of c multiplied by $b\ n$. The compression at any depth can be determined by proportion, as it varies as the width of the shaded triangle. Therefore, two-thirds from the neutral axis to the top, the compression will equal $\frac{2}{3}\ c$, half way up, $\frac{1}{2}\ c$, $\frac{1}{4}$ up, $\frac{1}{4}\ c$, etc. Or we may say :—The compression per square inch at any distance above the neutral axis will be to the maximum compression as the width of the triangle at that distance is to the width c. Now let c^1 equal the compression at a certain distance x above the axis, then $c^1 : c :: x : n$; hence $c^1 = \frac{c\ x}{n}$, which is a useful equation for future work.

The total tension will equal the tension per square inch in the steel multiplied by the sectional area of the steel ; it therefore equals $t\ a$.

The resistance of a beam to the external bending moment will be the total compression multiplied by the distance of its center from the neutral axis, plus the total tension multiplied by the distance of its center from the neutral axis. The center of compression will be at the center of gravity of the shaded triangle, Fig. 46 ; it equals $\frac{2}{3}\ n$ from the neutral axis. The center of tension is at a distance x', equal to $d—n$ from the neutral axis. Hence the full resisting moment equals :—

$$\frac{c\ b\ n}{2}\ \frac{2\ n}{3} + t\ a\ (d—n) = \frac{c\ b\ n^2}{3} + t\ a\ (d—n).$$

This, then, must equal the external bending moment M. It is not, however, a very convenient equation for designing, as it contains so many missing factors ; it greatly simplifies the calculations if we take the moments of resistance about a line through the center of the respective forces, i.e., through the center of the steel,

and $\frac{n}{3}$ from the compression surface ; the lever arms will then be equal, and, as the total compression equals the total tension, the moments of each will be equal, and each will also equal the external bending moment. This can be shown from the foregoing equation, as follows :—

$M = \frac{c\,b\,n}{2}\,\frac{2\,n}{3} + t\,a\,(d - n)$. Now as the total tension, $t\,a$,

equals the total compression $\frac{c\,b\,n}{2}$, we can say :

$M = \frac{c\,b\,n}{2}\,\frac{2\,n}{3} + \frac{c\,b\,n}{2}\,(d - n)$; which equals

$\frac{c\,b\,n}{2}\left(\frac{2\,n}{3} + d - n\right)$. Now, $\frac{2\,n}{3} + d - n = d - \frac{1}{3}\,n$; hence,

$M = \frac{c\,b\,n}{2}\left(d - \frac{n}{3}\right)$, and as $t\,a = \frac{c\,b\,n}{2}$ by substitution we

also get $M = t\,a\left(d - \frac{n}{3}\right)$. Therefore, the moment of

compression will be, $\frac{c\,b\,n}{2}\left(d - \frac{n}{3}\right)$, and the moment of

tension, $t\,a\left(d - \frac{n}{3}\right)$. Now to determine the strength of an existing beam we have to find the bending moment it will resist, and from this determine the load ; for this purpose we can use the equation :—

$M \cdot t\,a\left(d - \frac{n}{3}\right)$ or $M = \frac{c\,b\,n\left(d - \frac{n}{3}\right)}{2}$, but in either of these we have unknown factors, t, c, and n, two of which must be determined before we can use either equation.

In a rectangular beam of homogeneous material the neutral axis, n, is at half the depth, and the tension and compression are equal at equal distances from the axis. In such cases it is only necessary for us to fill in the maxi-

mum allowed value of either c or t, and we can at once determine the moment of resistance, which equals the bending moment ; but in a reinforced concrete beam we cannot do this as the axis is seldom at the half-depth, and owing to the steel being stronger than the concrete and its modulus of elasticity higher, it will require a much greater stress to stretch, or compress, it to the same extent as the concrete. Consequently, the stresses in the concrete and steel are not equal at equal distances from the axis ; the difference will depend on m, i.e., the ratio of

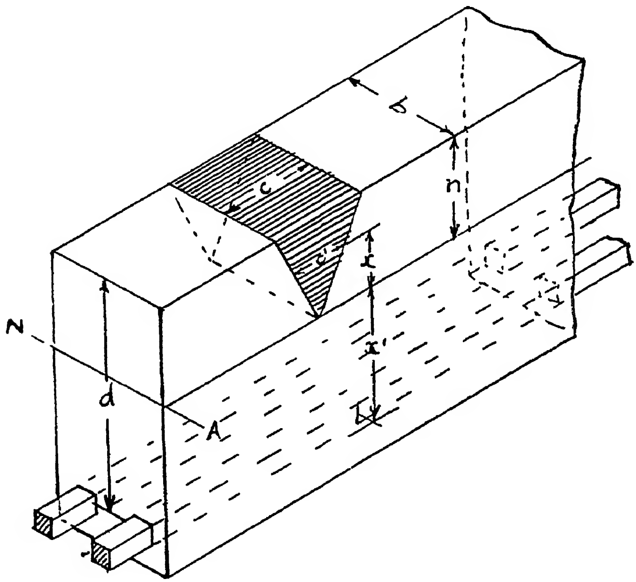


FIG. 46.

the moduli of elasticity of the concrete and steel ; see previous explanation.

Referring to Fig. 46, if x' equals n, the neutral axis will be at half the depth and the stress in the steel will be m times c. If x' equals $1\frac{1}{2}$ times n, the stress in the steel will be $1\frac{1}{2}$ times c multiplied by m. Hence, the stress in the steel will equal $\frac{x'}{n}$ times $c\,m$, and as x' equals $d-n$ we may say, the stress in the steel, t, equals $c\,m\left(\frac{d-n}{n}\right)$;

consequently, the stress in the concrete will be to the stress in the steel as the distance from the compressive surface to the neutral axis is to the distance from the steel to the neutral axis multiplied by the ratio of the moduli of elasticity of the materials. Thus :— $c : t :: n : (d—n)\, m$. From this we obtain the equation,

$$n = \frac{m\,c\,d}{t + m\,c} \text{ or } \frac{m\,c}{m\,c + t} = \frac{n}{d} \text{ (see chapter on formulas).}$$

Also knowing that the total tension, $t\,a$, is equal to the total compression, $\frac{b\,c\,n}{2}$, (see page 80). we can, from these equations determine the depth of the axis for any values of c, t, and m, and the ratio of area of steel to area of concrete required to develop these stresses in any beam. The ratio will equal the area of steel divided by the area of concrete, it equals p which equals $\frac{a}{b\,d}$.

Equating compression and tension, we get,

$$t\,a = \frac{b\,c\,n}{2}\,; \text{ therefore, } a = \frac{b\,c\,n}{2\,t}.$$

Now, by replacing a with its equivalent $\frac{b\,c\,n}{2\,t}$, we get,

$p = \frac{b\,c\,n}{2\,t\,b\,d}$ or $\frac{c\,n}{2\,t\,d}$, a convenient formula for use, and by which the values in the following table are obtained; they show the position of the neutral axis, and the tension in the steel that a given ratio of steel to concrete will develop with c at 600 lb., and what c will be with the same ratio of steel, but when it is stressed to 16,000 lb.

The corresponding values in the table read horizontally thus :—With a ratio of steel of 0·007,5, or 0·75 per cent., the axis is shown to be at a depth of 0·375 d, and when $c = 600$, $t = 15\,000$, or when $t = 17\,000$ the percentage of steel equals 0·61, and the axis is at a depth of 0·346 d, but if $t = 16\,000$, c will be only 572.

The values for any other stresses, not shown in the table, can be obtained as follows :—For $c = 500$ and $t = 14\,000$, $\frac{n}{d} = \frac{m\,c}{t + m\,c} = \frac{15 \times 500}{14\,000 + 15 \times 500} = 0{\cdot}348$, and $p = \frac{c\,n}{2\,t\,d}$. Now as $n = 0{\cdot}348\,d$, we can replace n with this equivalent and get, $p = \frac{0{\cdot}348\,d\,c}{2\,t\,d}$; by cancelling d, $p = \frac{0{\cdot}348\,c}{2\,t} = \frac{0{\cdot}348 \times 500}{2 \times 14\,000} = 0{\cdot}006\,2$.

When c = 600	m = 15		When t = 16 000
t =	$\frac{n}{d}$ =	p =	c =
9 000 = $m\,c$	0·5	0·016 66	1 066
10 000	0·473	0·014 14	960
11 000	0·45	0 012 27	872
12 000	0·428	0 010 7	800
13 000	0·409	0 009 43	736
14 000	0·391	0·008 37	685
15 000	0·375	0·007 5	640
16 000	0·36	0·006 75	600
17 000	0·346	0·006 1	572
18 000	0·333	0·005 55	533
19 000	0·321	0·005 22	504
20 000	0 31	0·004 65	479

From the above values it is evident it is not economical to use a high percentage of steel, for when it exceeds 0·675 per cent. it is not possible to stress it to its full value of 16,000 lb. per square inch without stressing the concrete over 600 lb. per square inch, and with a lower percentage it is not possible to stress the concrete to 600 lb. per square inch without stressing the steel over 16,000 lb. per square inch.

Figs 47 and 48 are simple diagrams, either of which can be used to determine n when c and t are fixed ; to determine c for any position of n when t is fixed, and to determine t for any position of n when c is fixed ; c, or

$c\ m$ may be set off along the top line, and t or $\frac{t}{m}$ along the bottom line; the diagonal will insect the vertical line at the neutral axis.

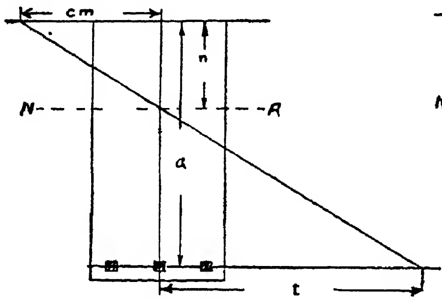


Fig. 47.

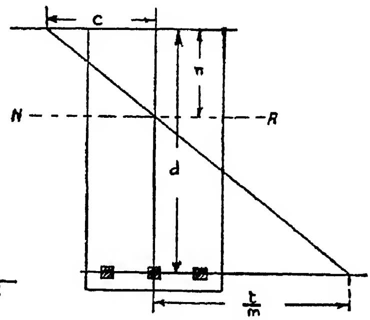


Fig. 48.

BENDING MOMENT AND SHEAR STRESSES.

For designing any portion of a structure acting as a beam, it is necessary to determine the value of the bending moments and shearing stresses which the member will have to resist. These will vary with the load, its distribution, and the manner in which the beam is fixed. Figs. 49 to 68 illustrate the principal cases, and give the values of the bending moments, shearing stresses, reactions of the supports, and diagrams illustrating the variation of the bending moments and shear stresses. In addition, the following particulars are useful :—On a cantilever, however loaded, the shearing stress at any point equals the total load between that point and the outer end, as in Fig. 50, where the shearing stress at $x = w\ l$, where w = the load per lineal foot. For Fig. 53, the shear at x equals the load on the portion

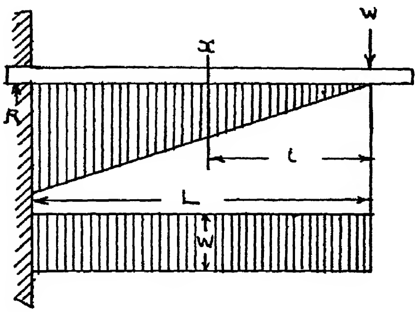
$$l = \frac{w\ l^2}{2\ L}.$$

For Figs. 51 and 52, the shear anywhere between the loads equals the load on the outer end.

In cantilevers, the bending moment at any point

equals the load outside of the point multiplied by the distance of its center of gravity from the point. In Fig. 50, M′ at x the load on the portion l multiplied by

$$\frac{l}{2} = \frac{w\,l^2}{2}.$$

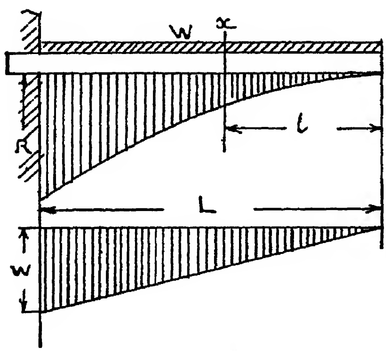


$$R = S = W$$

$$M = W\,L$$

$$M \text{ at } x = W\,l$$

FIG. 49.

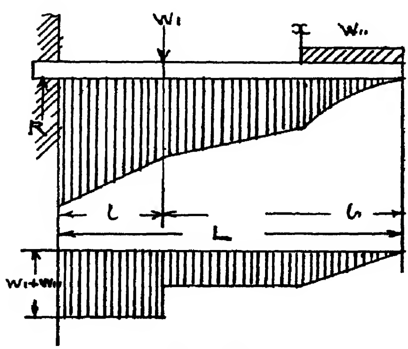


$$R = S = W$$

$$M = \frac{W\,L}{2}$$

$$M \text{ at } x = \frac{w\,l^2}{2}$$

FIG. 50.



$$R = S = W' + W''$$

$$M = W'l + W''\left(L - \frac{l'}{2}\right)$$

$$M \text{ at } x = \frac{W'\,l'}{2}$$

FIG. 51.

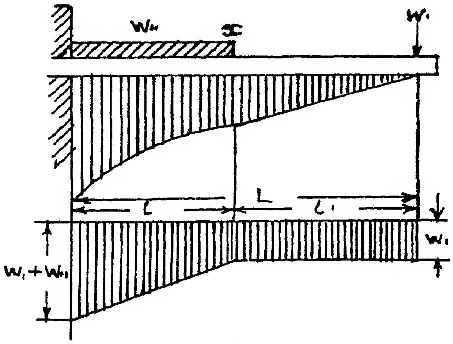


Fig 52.

$$R = S = W' + W''$$

$$M = W'L + W''\frac{l}{2}$$

$$M \text{ at } x = W'\,l'$$

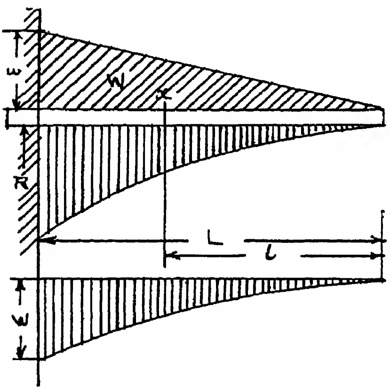


Fig. 53.

$R = S = W$

$W = w\,\dfrac{L}{2}$ Where w = load per foot at Wall.

$$M = \frac{W\,L}{3}$$

$$M \text{ at } x = \frac{w\,l^2}{3\,L}$$

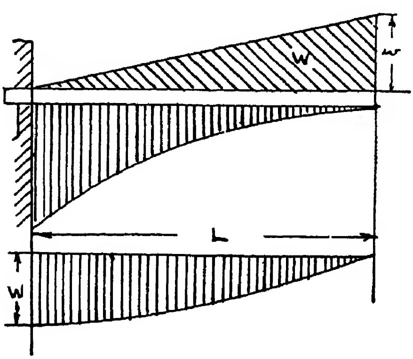


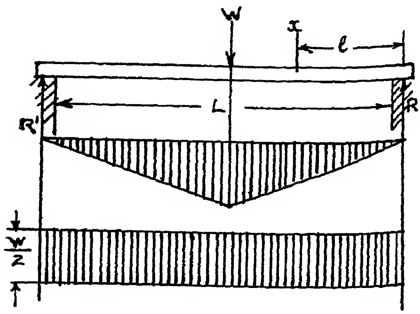
Fig. 54.

Load as Fig. 53 reversed.

$R = S = W$

$W = w\dfrac{L}{2}$

$M = \frac{2}{3}\,W\,L$

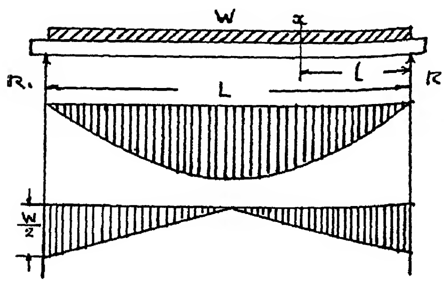


$$R = R' = S = \frac{W}{2}$$

$$M = \frac{W\,L}{4}$$

$$M \text{ at } x = R\,l$$

FIG. 55.

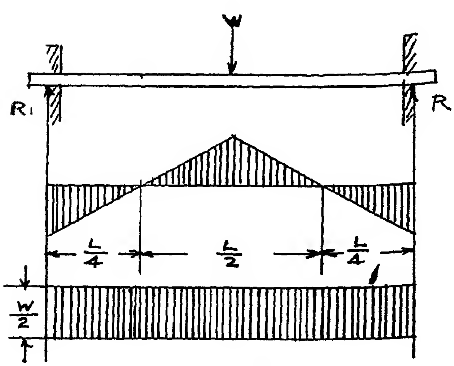


$$R = R' = S = \frac{W}{2}$$

$$M = \frac{W\,L}{8}$$

$$M \text{ at } x = R\,l - \frac{w\,l^2}{2}$$

FIG. 56



$R = R' = S = \frac{W}{2}$

$M = \frac{WL}{8}$ at center and ends.

FIG. 57.

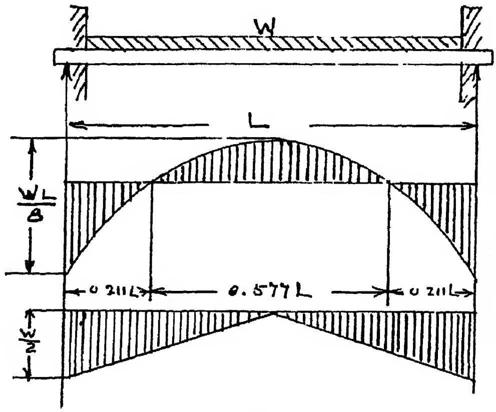


FIG. 58.

$$R = R' = S = \frac{W}{2}$$

$$M = \frac{W\,L}{12} \text{ at supports}$$

$$M = \frac{W\,L}{24} \text{ at center of}$$

Span.

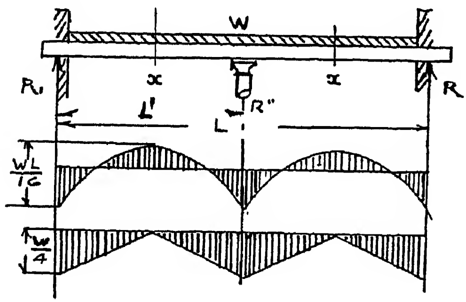


FIG. 59.

Each half as Fig. 58

or

$$R' = R = S = \frac{W}{4}.$$

$$R'' = \frac{W}{2}$$

$$M = \frac{W\,L}{24} \text{ at } R, R', R''$$

$$M = \frac{W\,L}{48} \text{ at } x$$

W = Load for whole Beam.

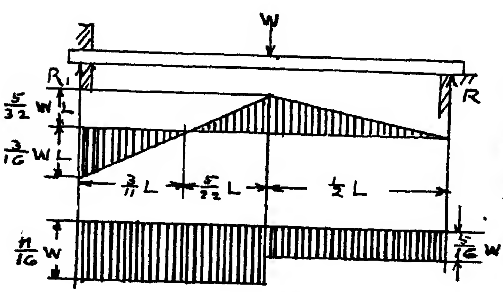


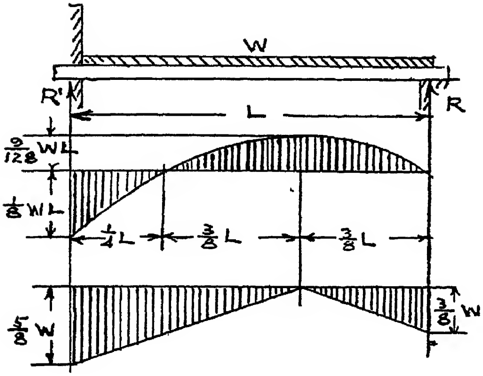
FIG. 60.

$R = \frac{5}{16}\,W = S'$

$R' = \frac{11}{16}\,W = S$

$M = \frac{5}{32}\,W\,L$ at center

$M = \frac{3}{16}\,W\,L$ at fixed end.



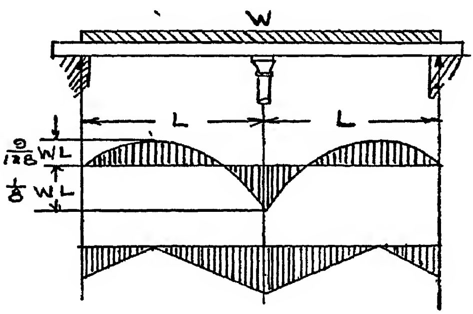
$R = \frac{3}{8} W = S$

$R' = \frac{5}{8} W = S$

$M = \dfrac{W\,L}{8}$ at fixed end

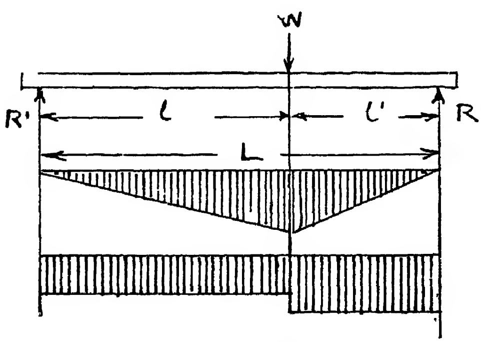
$M = \frac{9}{128}$ at $\frac{3}{8}$ L from supported end.

FIG. 61.



Each half as Fig. 61.

FIG. 62.



$R = \dfrac{W\,l}{L}\quad R' = \dfrac{W\,l'}{L}$

$M = R\,l' = R'\,l$

S on portion $l = R'$

S on portion $l' = R$

FIG. 63.

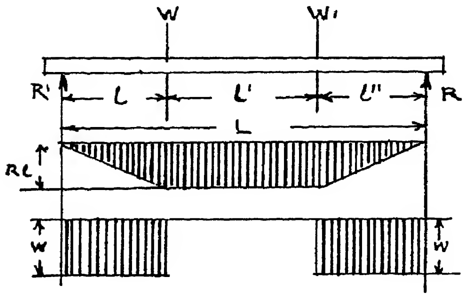


FIG. 64.

Where $l = l''$ and $W = W'$

$R = R' = S = W$

M between W & W' $= R\,l''$ or $R'\,l$

S on portion $l = R' = W$ or W'

S on portion $l' = 0$

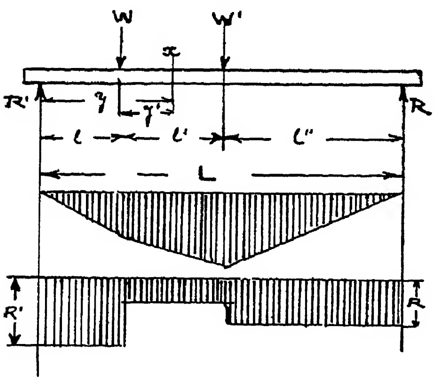


FIG. 65.

$$R = \frac{W'(l+l') + W\,l}{L}$$

$$R' = \frac{W'\,l'' + W(l+l'')}{L}$$

M at W $= R'\,l$

M at W' $= R\,l''$

or $R'(l+l') - W\,l'$

S at R $= R$

S at R' $= R'$

S between W & W' $= R' - W$

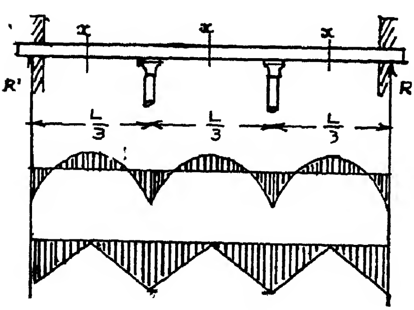


FIG. 66.

Each Span as Fig. 58. It imperfectly fixed

$$M \text{ at } x = \frac{W\,L}{12}$$

In Fig. 53 M′ at x = load on portion l multiplied by $\frac{l}{3} = \frac{w\,l^3}{6\,L}$ The moment at any other point, or for any other loading, can be obtained in a similar manner.

Figs. 53 and 54 represent cases analogous to earth and water pressures on walls, foundation slabs, etc., which are fully considered in Part II.

With beams, the sum of the reactions of the supports is equal to the total load. When both ends are alike supported or fixed, the portion of any load borne by either of the supports, is to the load as the further segment of the beam is to the whole length of the beam, thus :—referring to Fig. 63, R : W : : l : L ; therefore, $R = \frac{W\,l}{L}$, a formula by which we can determine the reaction for any load, l being the distance from the center of gravity of the load to the farther support.

The greatest shearing stress is equal to the greatest

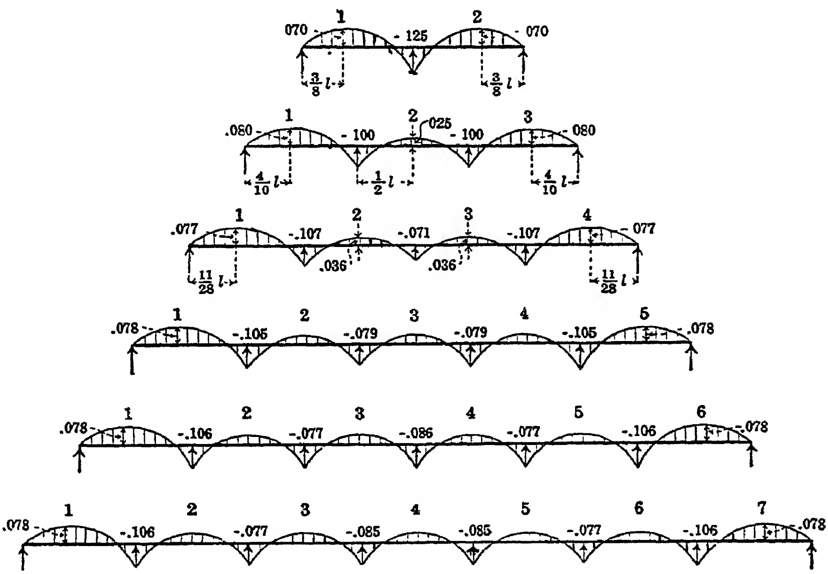


FIG. 67.

Moment co-efficients of W L for continuous beams of equal spans supported at the ends and uniformly loaded. W equals the load and L equals the length of the separate spans.

reaction. The shearing stress at any other point equals the nearest reaction minus all loads between that reaction and the point taken, as at x, Fig. 56, where the shearing stress equals R—$l\ w$; the bending moment at the same point will equal R l, minus the load on portion l multiplied by the distance of its center of gravity from x, $= Rl - \frac{w\ l^2}{2}$.

The above rule can be used to determine the bending moment at any point in any case of a beam freely supported at its ends. The rule may be expressed thus :— The bending moment at any point equals either reaction

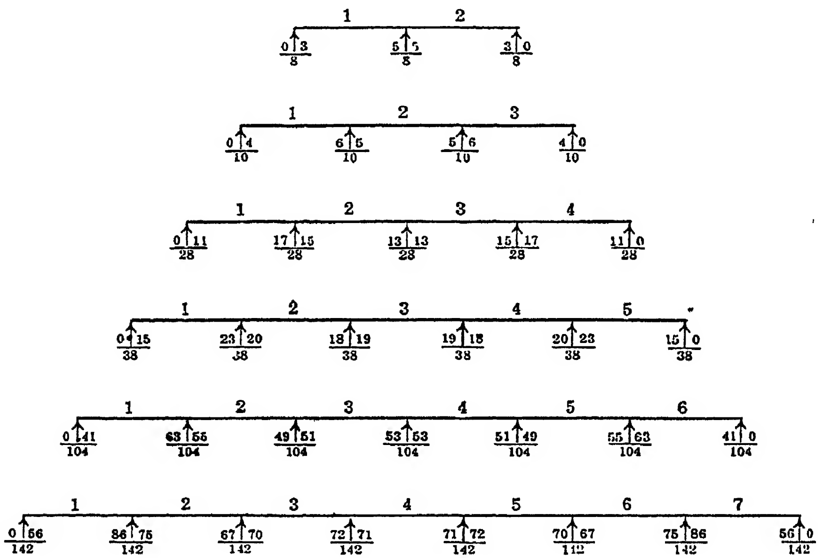


FIG 68.

Shear co-efficients of W L for continuous beams of equal spans supported at the ends and uniformly loaded. W equals the load and L equals the length of the separate spans.

multiplied by its distance to the point, minus any loads between the reaction taken and the point, multiplied by their distances from the point. The loads to be taken separately, and the distance of each measured from its center of gravity.

Referring to Fig. 65, as another example, the shearing stress at x = R′—W. The bending moment at the same point will be R′ y—W y'.

With beams securely fixed at their ends and extending over columns that are perfectly level with each other, and with the outside supports, each portion can be considered a fixed beam ; the bending moment over the columns and at the outside supports will be $\frac{W L}{12}$, and at the center of the spans $\frac{W L}{24}$, as Figs. 59 and 66. But owing to the practical impossibility of securing perfect fixing to the walls, it is advisable to take the bending moment for the end spans as $\frac{W L}{12}$, for both the ends and the center, for if the fixing is not absolutely perfect, the stress will be reduced at the supports, with a corresponding increase at the center of the span. For the same reason the bending moments for cases as Figs. 61 and 62 are generally taken as $\frac{W L}{10}$ at both critical points.

When a beam is supported by a beam, such as secondary beams on main beams, instead of on columns, as in Figs. 59 and 66, the bending moments should be taken as $\frac{W L}{12}$ for the center of the spans, and $\frac{W L}{24}$ over the supporting beams.

The following examples show the application of the formulas already given :

Example I.—The working stresses for the concrete and steel in a beam is to be 600 lb. and 16,000 lb. per square inch respectively. The ratio of the moduli of elasticity, m, equals 15. Determine the proportion of steel required to develop these stresses.

The proportion of steel, represented by p, equals $\frac{c\,n}{2\,t\,d}$. For this case, we know the values of c and t, but

n and d are unknown. We, however, know that $n = \frac{m\ c\ d}{t + m\ c}$, therefore, for these stresses, $n = \frac{15 \times 600d}{16\,000 + 15 \times 600} = 0 \cdot 36d$, where d equals the effective depth of the beam. Now by substituting the value of n in terms of d, we get :— $p = \frac{c \times 0 \cdot 36d}{2\ t\ d}$. By cancelling d, and filling in the values of c and t, $p = \frac{600 \times 0 \cdot 36}{2 \times 16\,000}$ $= 0 \cdot 006\,75$, as given in the table, page 85. Whatever the size of the beam the sectional area of the steel required to develop these stresses will be 0·006,75 the sectional area of the concrete. For instance :—for a beam 8 inches by 12 inches the steel would be $0 \cdot 006\,75 \times 8 \times 12 = 0 \cdot 648$ square inches.

Example II.—Determine the proportion of steel for a beam to be made of brick, or limestone concrete, for which the moduli ratio is 18, the maximum stress for the concrete, 500 lb. per square inch, and for the steel, 15,000 lb. per square inch.

Taking this in the same way as the case of Example I. we get :—

$$n = \frac{m\ c\ d}{t + m\ c} = \frac{18 \times 500\ d}{15\,000 + 18 \times 500} = 0 \cdot 375\,d.$$

And
$$p = \frac{c\ n}{2t\ d} = \frac{500 \times 0 \cdot 375\ d}{2 \times 15\,000\ d} = 0 \cdot 625.$$

To compare this case with that of Example I., a beam 8 inches by 12 inches would require $0 \cdot 625 \times 8 \times 12 = 0 \cdot 6$ square inches of steel; not so much as the beam of Example I., and it would not be so strong. If required to carry the same load as that of Example I. it would have to be increased in size, otherwise both concrete and steel would be overstressed.

Example III.—(a) Determine the reinforcement required for a beam 12 inches wide, 20 inches effective depth, i.e., 20 inches from the top to the center of the reinforcement, to enable the steel and the concrete to be stressed to 16,000 lb., and 600 lb. respectively at the same time.

(b) If the beam is 20 feet long, what uniformly distributed load would there be required to develop these stresses ?

(a) The sectional area of steel equals a, and as $p = \frac{a}{b\,d}$, see page 84, $a = p\,b\,d$. From Example I., $p =$ 0·006 75 ; therefore, $a = 0{\cdot}006\,75 \times 20 \times 12 = 1{\cdot}62$ square inches. Say 3 bars $\frac{3}{4}$ inch square.

(b) To determine the load we must first find the bending moment this beam will resist, then equate this with the external bending moment. As previously explained, the moment of tension, or the moment of compression, will equal the bending moment. The moment of tension equals, $t\,a\left(d - \frac{n}{3}\right)$. The external bending moment, M, for a uniformly distributed load, equals $\frac{\text{W L}}{8}$.

Therefore, $$\text{W} = \frac{8\,t\,a\left(d - \frac{n}{3}\right)}{\text{L}}.$$

From Example I., $n = 0{\cdot}36\ d = 0{\cdot}36 \times 20 = 7{\cdot}2$ inches. Filling in all the values we get :—

$$\text{W} = \frac{8 \times 16\,000 \times 1{\cdot}6\left(20 - \frac{7{\cdot}2}{3}\right)}{20 \times 12} = 15\,019 \text{ lb.}$$

Note that the external bending moment must be taken in the same terms as the internal bending moment. In reinforced concrete beams the depth, d, for the internal bending moment is always taken in inches ; consequently

the length for the external bending moment M must be taken in inches.

If a beam proportioned, as in this example, is too large for the position it is to occupy, it can be reduced, and compression bars added to replace the loss of concrete, and additional bars used to take the increase of tension. Such cases are considered further on.

Example IV.—Determine the load for a beam 18 inches effective depth, and 10 inches wide, over a span of 15 feet. The reinforcement consisting of 3 bars one-inch square. The stress in the concrete not to exceed 600 lb. per square inch, and in steel 16 000 lb. per square inch m to equal 15.

According to the previous example, the stresses of 600 ~~500~~ lb., and 16,000 lb. cannot exist at the same time with any load, unless the steel is 0·006 75bd. If more steel than this, with the concrete stressed to 600 lb., the steel will be understressed, or with the steel stressed to 16,000 lb., the concrete will be overstressed. This enables us to decide whether to use the moment of tension or of compression to determine the external bending moment M, which we must have to determine the load.

The proportion of steel to concrete equals

$$p = \frac{a}{b\,d}, \; a = 3, \; b = 10, \; d = 18\,;$$

therefore;
$$p = \frac{10 \times 18}{3} = 0{\cdot}016\,6\,;$$

consequently, being more than 0·006,75, the critical percentage, we must determine the bending moment, M, by the moment of compression, which equals $\dfrac{c\,b\,n\left(d - \frac{n}{3}\right)}{2}$.

But before working this out we have to determine n. The equation, $n = \dfrac{m\,c\,d}{t + m\,c}$, cannot be used in this case as we do not know both c and t; we can, however, fix

one of these ; the other will depend on the position of n. By referring to the values on page 85, we find where $p = 0{\cdot}016{,}6$, $n = 0{\cdot}5\ d$; hence, for this case n will $0{\cdot}5 \times 18 = 9$ inches.

Taking $c = 600$, and filling in the other values, we get :

$$M = \frac{600 \times 10 \times 9 \left(18 - \frac{9}{3}\right)}{2} = 405\ 000 \text{ inch lb.}$$

The tension in the steel, from page 85 will be 9,000 lb. per square inch. In the absence of the table it can be determined by the diagram, Fig. 47, or by equating compression and tension, thus :— $t\,a = \frac{c\,b\,n}{2}$; therefore, $t = \frac{c\ b\ n}{2\ a} = \frac{600 \times 10 \times 9}{2 \times 3} = 9{,}000$ lb. Or again, by equation, page 83, where $t = c\ m \left(\frac{d-n}{n}\right)$.

In the above example, n turned out to agree with one of the values in the table, page 85. If it had not done so, or in the absence of a table, one of the following formulas could be used.

1. When the steel is expressed in inches.

$$n = \sqrt{\frac{2\ m\ a\ d}{b} + \left(\frac{m\ a}{b}\right)^2} - \frac{m\ a}{b}, \text{ or further simplified,}$$

$$n = \frac{\sqrt{2\ m\ a\ d\ b + (m\ a)^2} - m\ a}{b}.$$

2. When the steel is expressed in terms of the area of concrete, as p. $n = d[\sqrt{2\ p\ m + (p\ m)^2} - p\ m]$. The derivations of these are given in the chapter on formulas.

Example V.—Determine the distributed load for a granite concrete beam, effective depth 24 inches, breadth 15 inches, reinforced with 4 bars $\frac{3}{4}$ inch square, over an effective span of 24 feet. The stress in the concrete is not to exceed 600 lb. per square inch, and in the steel 16,000 lb. per square inch.

This case and that of Example IV. are similar. They are given to get more acquainted with the principles, and the method of working such cases.

The load must be determined as in the previous case, by equating the bending moment with the moment of resistance, and by substituting the value of the bending moment in terms of the load and length of the beam but whether to use the moment of compression or of tension, depends upon the proportion of steel, as explained in Example IV. It equals $\frac{a}{b\,d}$, $a = 4$ bars $\frac{3}{4}$ inch square, $= 4 \times 0{\cdot}5625$, $b = 15$, $d = 24$;

therefore $$p = \frac{4 \times 0{\cdot}562\,5}{15 \times \cancel{25}\,24} = 0{\cdot}006\,25.$$

Which is less than the economic proportion of 0·00675. We must, therefore, use the moment of tension to determine the bending moment, see previous example. The moment of tension equals $t\,a\left(d - \frac{n}{3}\right)$. The bending moment, M, equals $\frac{W\ L}{8}$

Equating these two we get:— $$\frac{W\,L}{8} = t\,a\left(d - \frac{n}{3}\right).$$

Hence $$W = \frac{8\,t\,a\left(d - \frac{n}{3}\right)}{L},$$

all of which, except n, are known. For this proportion of steel, n is not given in the table, page 85; we

must, therefore, determine its value from the formula,

$$n = \sqrt{\frac{2\,m\,a\,d}{b} + \left(\frac{m\,a}{b}\right)^2} - \frac{m\,a}{b}.$$

For granite concrete, $m = 15$; hence

$$n = \sqrt{\frac{2 \times 15 \times 2{\cdot}25 \times 24}{15} + \left(\frac{15 \times 2.25}{15}\right)^2} - \frac{15 \times 2{\cdot}25}{15}$$

$$= \sqrt{10{\cdot}63} - 2{\cdot}25 = 8{\cdot}38.$$

Now filling in the values we get :

$$W = \frac{8 \times 16\,000 \times 2{\cdot}25\left(24 - \frac{8{\cdot}35}{3}\right)}{24 \times 12} = 21217 \text{ lb.}$$

The stress in the concrete can now be determined, by knowing that the total tension equals the total compression, as previously explained ;

therefore $$t\,a = \frac{c\,b\,n}{2},$$

hence $$c = \frac{2\,t\,a}{b\,n} = \frac{2 \times 16\,000 \times 2{\cdot}25}{15 \times 8{\cdot}38} = 572 \text{ lb.}$$

This can also be obtained by either formula 13 or 14 in the chapter on formulas.

Example VI.—Compare the load for the beam of Example V. with a beam of the same size, but made of brick concrete, for which the safe stress is 500 lb. per square inch, and $m = 18$.

From Example V. we get :—$p = 0{\cdot}006,25$. Proceeding as before, we have first to determine whether to use the moment of tension or of compression to equate with

the bending moment to find the load. For stresses of 500 and 16,000, with $m = 18$, the economic percentage of steel will not be 0·006,75, as for the case of Example V.; we therefore cannot tell by that example which moment to use; we can determine the load for the concrete stress and for the steel separately, then adopt the smaller, but this is not a very systematic method; it is better to proceed as explained in the previous example.

The critical percentage for these stresses, with $m = 18$, is not given in the table; we can, however, find it by the formula explained on page 84, where $p = \dfrac{c\,n}{2\,t\,d}$ and $n = \dfrac{m\,c\,d}{\substack{\cancel{t m c} \\ t + mc}}$. $m = 18$, $c = 500$, $t = 16\,000$;

hence,
$$n = \frac{18 \times 500 \times 24}{18 \times 500 + 16\,000} = 8{\cdot}64,$$

and
$$p = \frac{500 \times 8{\cdot}64}{2 \times 16\,000 \times 24} = 0{\cdot}005\,62.$$

From this we see that the actual steel in the beam is more than the critical amount; we must, therefore, determine the load from the moment of compression, the reverse to that of Example V.

The moment of compression equals $\dfrac{c\,b\,n\left(d - \dfrac{n}{3}\right)}{2}$.

The bending moment, M, equals $\dfrac{W\,L}{8}$. Equating these we get :—

$$W = \frac{8\,c\,b\,n\left(d - \dfrac{n}{3}\right)}{2\,L}$$

$$\frac{8 \times 500 \times 15 \times 8{\cdot}64\left(24 - \dfrac{8{\cdot}64}{3}\right)}{2 \times 24 \times 12} = 19\,008 \text{ lb.}$$

From this we see that if the beam is made of broken brick, average quality limestone, or furnace slag concrete, it will carry only 87 per cent. of the load that a granite concrete beam will carry.

Example VII.—Design a beam 20 feet effective span, supported at the ends, to carry a distributed load of 500 lb. per lineal foot, including its own weight.

In designing beams many designers make it a rule to first assume the dimensions as near as their experience enables them, and then work out the stresses and determine their sufficiency or otherwise. This, however, is not a very satisfactory method, especially for reinforced concrete, for it may involve several trials, and then have only a fairly satisfactory result, for these beams should be accurately proportioned, so as to allow both the concrete and steel to be stressed to their maximum allowance, we will then have the most economical section, for there will be no waste of concrete or steel.

By referring to page 85, when $c = 600$, $t = 16\ 000$, and $m = 15$, n is shown to be $0{\cdot}36\ d$, and $p = 0{\cdot}006\ 75$.

For this case, we have, therefore, to design a beam containing 0·675 per cent. area of steel to area of concrete, the latter being sufficient to resist the compression. This may be done with little trouble by using the equation, $M = 95\ b\ d^2$, given with its derivation in the chapter on formulas. By this we can determine b and d, then make the area of steel equal $p = 0{\cdot}006\ 75\ b\ d$, b may be taken as $\frac{1}{20}$ to $\frac{1}{24}$ of the span,

then $$d = \sqrt{\frac{M}{95\ b}},$$

or d may be taken as $\frac{1}{12}$ to $\frac{1}{16}$ of the span,

then $$b = \frac{M}{95\ d^2}.$$

It is, however, much better to make b some definite proportion of d, say from 0·5 to 0·7 of d, according to circumstances, 0·6 d is usually the most suitable proportion for a reinforced beam. Under these conditions the

case may be worked as follows :—Substituting the value of b in terms of d, as $0{\cdot}6\ d$, then $M = 95 \times 0{\cdot}6d \times d^2 = 95 \times 0{\cdot}6\ d^3$;

hence $$d = \sqrt[3]{\frac{M}{95 \times 0{\cdot}6}}$$

and $$M = \frac{W\ L}{8} = \frac{500 \times 20 \times 20 \times 12}{8} = 300\ 000 \text{ inch lb.}$$

Therefore, $$d = \sqrt[3]{\frac{300\ 000}{95 \times 0{\cdot}6}} = \sqrt[3]{5\ 263} = 17{\cdot}4 \text{ inches.}$$

And $a = p\ b\ d = 0{\cdot}006\ 75 \times 10{\cdot}44 \times 17{\cdot}4 = 1.23$ square inches.

The nearest section to give this will be three $\frac{3}{4}$ inch round bars, $= 1{\cdot}32$ square inches ; or four $\frac{5}{8}$ inch square. Three $\frac{11}{16}$ inch square bars, giving $1{\cdot}42$ square inches, would be nearer than the $\frac{5}{8}$-inch, but some factories do not make bars rising in sixteenths of an inch, so these should not be specified unless first finding if they are obtainable.

The effective section will therefore be, $17\frac{1}{2}$ inches by $10\frac{1}{2}$ inches with, say, 3 bars $\frac{3}{4}$ inch square.

Allowing 2 inches of concrete to cover the bars, the full depth will be $19\frac{1}{2}$ inches.

To complete the case, the shear and adhesive stresses must be considered ; these are explained further on.

Example VIII.—Design a beam to carry a distributed load of 18,000 lb. ; exclusive of its own weight, over a span of 24 feet.

The total load will be 18,000 lb. plus the weight of the beam ; this cannot be accurately determined, as the dimensions are not known, but an approximation can be made, and if found to vary much from the actual amount the load can be adjusted accordingly, and the case worked again. A near approximation will be obtained

by assuming the depth to be about $\frac{1}{12}$ of the span and the breadth about $\frac{1}{20}$ of the span, say 24 inches by 15 inches.

At 150 lb. per cubic foot, the approximate weight will be, $2 \times 1 \cdot 25 \times 24 \times 150 = 9{,}000$ lb. Then the total load for the beam will equal $9{,}000 + 18{,}000 = 27{,}000$ lb. Working in a similar manner to Example VII., we get :—

$$M = \frac{W\ L}{8} = \frac{27\ 000 \times 24 \times 12}{8} = 972\ 000 \text{ lb.}$$

For the maximum stress in the concrete and steel to be 600 lb., and 16,000 lb. respectively, $M = 95\ b\ d^2$.

Taking b as $0 \cdot 6\ d$, then

$$d = \sqrt[3]{\frac{M}{95 \times 0 \cdot 6}} = \sqrt[3]{\frac{972\ 000}{95 \times 0 \cdot 6}} = 25 \cdot 8 \text{ inches.}$$

$b = 0 \cdot 6 \quad d = 0 \cdot 6 \times 25 \cdot 8 = 15 \cdot 48$ inches. With these stresses $p = 0 \cdot 006{,}75$, see page 00. Therefore $a = 0 \cdot 006{,}75 \quad b \quad d = 0 \cdot 006{,}75 \times 15 \cdot 48 \times 25 \cdot 8 = 2 \cdot 7$ square inches. Say 4 bars $\frac{7}{8}$ inch square $= 3$ square inches.

Allowing 2 inches of concrete to cover the bars, the full section will be 28 inches by $15\frac{1}{2}$ inches, a trifle more than allowed for in estimating the weight, but not enough to make any appreciable difference in the stresses.

HOW TO CHECK EXISTING SINGLY REINFORCED BEAMS.

To determine the stresses in any beam with single reinforcement proceed as follows :—

(1) Determine the total load, including the weight of the beam.

(2) Determine the bending moment.

(3) Determine the position of the neutral axis.

(4) Determine the maximum compression in the concrete, by the formula,

$$c = \frac{2\ M}{b\,n\left(d - \frac{n}{3}\right)}$$

(5) Determine the tension in the steel by one of the following formulas :—

(a) $$t = \frac{c\,m\,(d-n)}{n}$$

(b) $$t = \frac{M}{a\left(d-\frac{n}{3}\right)}$$

(c) $$t = \frac{c\,b\,n}{2\,a}$$

Example IX.—Determine the stresses in a beam supported at its ends and loaded with 1,000 lb. per lineal foot. Effective span 28 feet, full depth 30 inches, effective depth 28 inches, breadth 16 inches, reinforcement consisting of three rods $1\frac{1}{2}$ inch square.

(1) Load on beam $=1{,}000 \times 28 = 28{,}000$ lb.

Weight of beam $= \frac{30}{12} \times \frac{16}{12} \times 28 \times 150$ lb. $= 14\,000$ lb.

Total load $= 28\,000 + 14{,}000 = 42\,000$ lb.

(2) $$M = \frac{W\,L}{8} = \frac{42\,000 \times 28 \times 12}{8} = 1\,764\,000 \text{ inch-lb.}$$

(3) $$n = \sqrt{\frac{2\,m\,a\,d}{b} + \left(\frac{m\,a}{b}\right)^2} - \frac{m\,a}{b}. \quad m = 15, \quad d = 28,$$

$$b = 16, \quad a = 3 \times 1{\cdot}5^2 = 6.75.$$

Hence,

$$n = \sqrt{\frac{2 \times 15 \times 6{\cdot}75 \times 28}{16} + \left(\frac{15 \times 6{\cdot}75}{16}\right)^2} - \frac{15 \times 6{\cdot}75}{16} = 13{\cdot}53.$$

(4) $$c = \frac{2\,M}{b\,n\left(d-\frac{n}{3}\right)} = \frac{2 \times 1\,764\,000}{16 \times 13{\cdot}53\left(28 - \frac{13{\cdot}53}{3}\right)} = 694 \text{ lb.}$$

$$(5)\quad t = \frac{c\ m\ (d-n)}{n} = \frac{694 \times 15\ (28-13 \cdot 53)}{13 \cdot 53} = 11\ 133 \text{ lb.}$$

The shear stress also should be checked; this is explained further on.

Example X.—Determine the dimensions and the reinforcement for a cantilever, which is to project 6 feet from its support, and is to carry a load of 1,000 lb. per lineal foot. The stresses are not to exceed 600 lb. and 16 000 lb. respectively for the concrete and steel, with $m = 15$.

Allowing 100 lb. per lineal foot for the weight of the cantilever, the total load will be $1{,}100 \times 6 = 6{,}600$ lb.

For a cantilever with a distributed load,

$$M = \frac{W\ L}{2} = \frac{6\ 600 \times 6 \times 12}{2} = 237{,}600 \text{ inch-lb.}$$

Making the breadth uniform, and equal to half the depth at the support, d, at the support will equal

$$\sqrt[3]{\frac{M}{95 \times 0 \cdot 5}} = \sqrt[3]{\frac{237\ 600}{95 \times 0 \cdot 5}}.$$

Say 17 inches; for explanation of this formula refer to Example VII., $b = 0 \cdot 5\ d = 8\frac{1}{2}$ inches. $a = p\ b\ d = 0 \cdot 006\ 75 \times 8 \cdot 5 \times 17 = 0 \cdot 975$ square in.

For this, being a cantilever, the bending moment will diminish towards the end; therefore, if desirable, the size of the cantilever can diminish also. The bending moment at the center of the length will equal $\frac{w\ l^2}{2}$, see Fig. 50, $= \frac{1\ 100 \times 3^2 \times 12}{2} = 59{,}400$ inch-lb.

Keeping the breadth $8\frac{1}{2}$ inches throughout, from the formula, M = 95 $b\ d^2$, the effective depth at the center will equal

$$\sqrt{\frac{M}{95\ b}} = \sqrt{\frac{59\ 400}{95 \times 8{\cdot}5}};$$

say $8\frac{1}{2}$ inches, which is half the effective depth at the support. The reinforcement at the center will equal $p\ b\ d = 0{\cdot}006{,}75 \times 8.5 \times 8{\cdot}5 = 0{\cdot}488$ square inch. Allowing for a depth of 4 inches at the end, and 2 inches

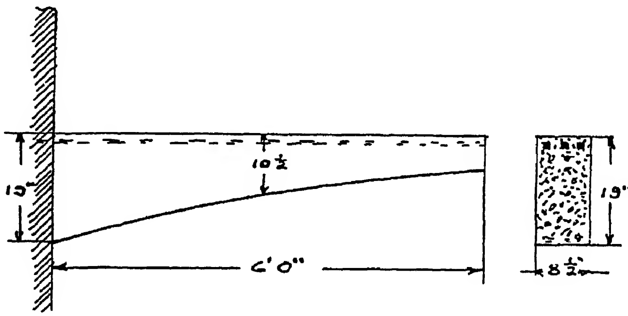


FIG. 69.

cover to the steel throughout the whole length, the design will be as Fig. 69, with two $\frac{5}{8}$ inch square rods running the whole length along the top, and one extra for the inner half.

SINGLE REINFORCED BEAMS, LARGER OR SMALLER THAN THE ECONOMIC SECTION.

It is sometimes necessary for us to employ a beam that is larger, or smaller, than the size necessary to form what we know as the economic section for a given load. In such cases, the concrete would be either understressed or overstressed if the economic percentage of steel were used. This would be the case when the bending moment M is greater, or less, than the moment of compression, $\frac{c\ b\ n\left(d-\frac{n}{3}\right)}{2}$, in which $n = \frac{m\ c\ d}{t+m\ c}$; or when $b\ d^2$ is greater

or less than $\dfrac{6\,d\,M}{c\,n\left(3-\dfrac{n}{d}\right)}$, which, when $c = 600$, and $t = 16\,000$, equals $\dfrac{95}{M}$. $\dfrac{M}{95}$

Case A. When M is less than $\dfrac{c\,b\,n\left(d-\dfrac{n}{3}\right)}{2}$ or when, $b\,d^2$ is greater than $\dfrac{6\,d\,M}{c\,n.\left(3-\dfrac{n}{d}\right)}$, if the economic percentage of steel were used, both concrete and steel would be understressed ; we can therefore economise by using a lower percentage of steel ; this would raise the axis of the beam, through the steel being higher stressed, and would thus reduce the compression area causing the concrete to be higher stressed, but not overstressed, as the steel is less than the economic percentage. The most economical amount of steel for such cases will be that which will permit it to be stressed to its limit.

Case B. When M is greater than $\dfrac{c\,b\,n\left(d-\dfrac{n}{3}\right)}{2}$, or when $b\,d^2$ is less $\dfrac{6\,d\,M}{c\,n\left(3-\dfrac{n}{d}\right)}$, if the economic percentage of steel were used both concrete and steel would be overstressed. To permit both concrete and steel to be stressed to their respective limits compression bars may be added, as shown under beams with double reinforcement. If these are not permissible, but single reinforcement only, we can stress the concrete to its limit, without overstressing the steel, only by employing more than the economic percentage of steel, which will itself be understressed, but will lower the axis of the beam sufficiently to provide a compression area large enough to take all

the compression when the maximum equals the value given to c. This can be done by the formulas in Example XI., providing the depth of the beam is within a reasonable limit.

Examples XI. and XII. are given to elucidate the foregoing explanation.

Example XI.—Determine the steel required for a singly reinforced beam 26 inches effective depth, 15 inches wide, to carry a load of 25,000 lb., including its own weight, over a span of 20 feet. The stresses for the concrete and steel not to exceed 600 lb., and 16,000 lb. respectively, with $m = 15$.

This is a case similar to that under A.

$$M = \frac{W\,L}{8} = \frac{25\,000 \times 20 \times 12}{8} = 750\,000 \text{ inch-lb.}$$

For the economic section, $b\,d^2$ equals $\frac{M}{95}$; if greater than this we can use less steel than $0{\cdot}006\,75\;b\,d$, if smaller we must use more than this.

$$\frac{M}{95} = \frac{750\,000}{95} = 7\,895. \quad b\,d^2 = 15 \times 26^2 = 10{,}140\;;$$

therefore, we can use less steel than $0{\cdot}006{,}75\;b\,d$. The reduced amount can be determined by the formula,

$a = \frac{M}{0{\cdot}88\,t\,d,}$ in which t is given its maximum value, in this case 16 000. Filling in the values we get :—

$$a = \frac{750\,000}{0{\cdot}88 \times 16\,000 \times 26} = 2{\cdot}048 \text{ square inches.}$$

Determining the stress in the concrete by the formula,

$c = \frac{2\,t\,a}{b\,n}$, and $n = \sqrt{\frac{2\,m\,a\,d}{b} + \left(\frac{m\,a}{b}\right)^2} - \frac{m\,a,}{b}$ we get :—

$$n = \sqrt{\frac{2 \times 15 \times 2{\cdot}048 \times 26}{15} + \left(\frac{15 \times 2{\cdot}048}{15}\right)^2} - \frac{15 \times 2{\cdot}048}{15} = 8{\cdot}474 \text{ in.}$$

Hence $\quad c = \dfrac{2 \times 16\,000 \times 2{\cdot}048}{15 \times 8{\cdot}474} = 515$ lb.

Example XII.—Determine the steel required for a singly reinforced beam 20 inches effective depth, 12 inches wide, to resist a bending moment of 700,000 inch-lb. The stress in the concrete not to exceed 600 lb. per square inch, with $m = 15$.

This is a case similar to that under B.

Proceeding as in the last example, $\dfrac{\text{M}}{95} = \dfrac{700\,000}{95} =$ 7 368. $b\,d^2 = 12 \times 20^2 = 4{,}800$. The latter, being the smaller quantity, shows that the beam is smaller than what is known as the economic section ; it will therefore, be necessary to employ more steel, for single reinforcement, then the economic proportion of 0·006,75 $b\,d$. The correct amount can be determined by the formula :—

$$a = \frac{b\,n^2}{2\,m\,(d - n)}, \text{ and } n = \frac{3\,d}{2} - \sqrt{\left(\frac{3\,d}{2}\right)^2 - \frac{6\,\text{M}}{c\,b}}.$$

For the derivation of these, refer to the chapter on formulas. Filling in the values for the neutral axis we get :

$$n = \frac{3 \times 20}{2} - \sqrt{\left(\frac{3 \times 20}{2}\right)^2 - \frac{6 \times 700\,000}{600 \times 12}} = 12{\cdot}21.$$

Hence, $\quad a = \dfrac{12 \times 12{\cdot}21^2}{2 \times 15\,(20 - 12{\cdot}21)} = 7{\cdot}655.$

The stress in the steel will equal

$$t = \frac{c\,b\,n}{2\,a} = \frac{600 \times 12 \times 12{\cdot}21}{2 \times 7{\cdot}655} = 5{,}742 \text{ lb.};$$

which is about 35 per cent. of the limiting steel stress, but if the steel is reduced to enable it to be stressed higher the concrete would be overstressed.

As now we have the quantity of steel we can check the correctness of the formulas used, by the usual method, where,

$$n = \sqrt{\frac{2\,m\,a\,d}{b} + \left(\frac{m\,a}{b}\right)^2} - \frac{m\,a}{b}, \text{ and } c = \frac{2\text{M}}{b\,n\left(d - \frac{n}{3}\right)}.$$

Filling in the values for these we get :—

$$n = \sqrt{\frac{2 \times 15 \times 7{\cdot}655 \times 20}{12} + \left(\frac{15 \times 7{\cdot}655}{12}\right)^2} - \frac{15 \times 7{\cdot}655}{12} = 12{\cdot}21.$$

$$c = \frac{2 \times 700\ 000}{12 \times 12{\cdot}21\left(20 - \frac{12{\cdot}21}{3}\right)} = 600 \text{ lb.}$$

If the depth of the beam was not limited the economic section could be adopted, where, for $c = 600$, and $t = 16\ 000$, $d = \sqrt{\frac{\text{M}}{95\ b}}$, and $a = 0{\cdot}006\ 75\ b\ d$.

By these we get :

$$d = \sqrt{\frac{700\ 000}{95 \times 12}} = 24{\cdot}78 \text{ inch},$$

and $a = 0{\cdot}006\ 75 \times 12 \times 24{\cdot}78 = 2$ square inches.

From this we see that for an extra depth of 4·78 inches, an increase of about 58 square inches in the section of the beam, we would save 5·655 square inches of steel, this, with the concrete at 45 cents a cubic foot, and the steel at 5 cents a pound, would effect a saving of about 75 cents per lineal foot of beam.

BEAMS WITH DOUBLE REINFORCEMENT.

From the foregoing examples it can be seen that a beam with single reinforcement proportioned, as Examples VII. and VIII., to allow the concrete and steel to be stressed to their maximum allowance, will be rather deep in relation to the span, or very wide in relation to the depth ; for if the depth is kept down, the breadth must rapidly increase for the stresses to remain the same. For instance, with the beam of Example VIII., if the effective depth be reduced to 22 inches, it would require a breadth of 22 inches for the same load and stresses. The section would thus contain 84 square inches of concrete more than the one designed ; it is therefore wasteful to reduce the depth and increase the breadth. If circumstances prevent a beam properly proportioned being used, or if one, or both, of the dimensions are limited to a smaller amount, compression bars can be used to take up the excess of stress over that which the concrete is capable of resisting, thus forming a doubly reinforced beam. The section of concrete being less than for the economic section, the shear stress per square inch will be greater, and probably sufficient to require heavy shear members ; explained further on.

Beams with double reinforcement can be designed by several different methods, depending upon various conditions, which are explained with the following examples. With the most general method, which can be used for any case, it is usual to proceed in the following order :—

(a) Decide on the breadth and depth as near to the economic section as circumstances will permit.

(b) Determine the bending moment.

(c) Consider the beam as a single reinforced beam, and add reinforcement, as in Examples VII. and VIII., i.e., such as will develop the allowed stresses in a beam of that section by making $a = p\,b\,d$, p can be taken from the table, page 85, or determined by the equation

$$p = \frac{c\,n}{2\,t\,d}.$$

(d) Determine the bending moment that this section of concrete and steel will resist, irrespective of the actual bending moment the completed beam will have to resist.

(e) Add compression bars, and additional tension bars, to take up the excess of the bending moment, i.e., the difference between the moment for the single reinforced beam and the actual moment. This can be done by equating this excess bending moment with the moments of compression and tension of the additional reinforcement, as in the following example.

For other methods see further examples.

Example XIII.—Taking the case of Examples VIII., and limiting the effective depth to 22 inches, and the breadth to 14 inches.

Allowing two inches of concrete to cover the bars, the full depth will be 24 inches, and the total load will be 18,000 lb. plus the weight of the beam ; it will therefore

$$\text{equal } 18\,000 + 2 \times \frac{14}{12} \times 24 \times 150 = 26\,400 \text{ lb.}$$

$$M = \frac{W\,L}{8} = \frac{26\,400 \times 24 \times 12}{8} = 950\,400 \text{ inch-lb.}$$

When $c = 600$, and $t = 16\,000$, a will equal $0{\cdot}006\,75\; b\,d$, (see page 85). $0{\cdot}006\,75 \times 22 \times 14 = 2{\cdot}079$ square inches. The bending moment this will resist will equal the moment of compression, or of tension, thus :—

By the moment of tension,

$$\mathrm{M} = t\,a\left(d - \frac{n}{3}\right)$$

and $$n = \frac{m\,c\,d}{t + m\,c} = \frac{15 \times 600 \times 22}{16\,000 + 15 \times 600} = 7{\cdot}92,$$

or from the table, page 85, $n = 0{\cdot}36\,d$.

Therefore

$$\mathrm{M} = 16\,000 \times 2{\cdot}079\left(22 - \frac{7{\cdot}92}{3}\right) = 643\,991 \text{ lb.},$$

leaving a balance for additional reinforcement of 950 400 − 643 991 = 306 409 lb.

Now consider we have an extra beam consisting only of tension and compression bars, which take all the stress from the excess bending moment; we can then determine the section of steel required by taking moments about the center of compression and tension, as before; but in this case the center of compression will be at the center of the compression bars, which bars may be placed anywhere within the compression area, but should be as far as possible from the neutral axis; 2 inches from the top is a convenient position in beams.

The moment of tension will then be, $t\,a'\,(d—y)$, where a' equals the additional steel in tension, and y equals the distance of the compression bars from the top of the beam.

The moment of compression will be, $A_c\,c'\,(d—y)$, where A_c = area of steel in compression, and c', the compression per square inch in the steel.

Both of these moments must equal the excess bending moment; therefore, $t\,a'\,(d—y)$ = excess M;

hence, $$a' = \frac{\text{excess M}}{t\,(d—y)}.$$

By placing this additional tension steel with that already determined, t will be the same; consequently,

taking y as 2 inches,

$$a' = \frac{306\,409}{16\,000\,(22-2)} = 0{\cdot}957\,5 \text{ square inch.}$$

Similarly, the compressive steel,

$$A_c = \frac{\text{excess M}}{c'\,(d-y)}.$$

Now c' is really fixed as it will depend on the value of m, and on the stress in the concrete at the point where the bars are placed, in this case 2 inches from the top. c' will then equal m times the stress in the concrete 2 inches from the top; see page 83; hence it will be $m\,c\,\frac{n-y}{n}$; $\frac{n-y}{n}$ being the ratio of the maximum stress for the point where the bars are placed; therefore,

$$A_c = \frac{\text{excess M}}{m\,c\,\frac{n-y}{n}\,(d-y)}$$

and n, already obtained, $= 7{\cdot}92$; hence,

$$A_c = \frac{306\,409}{15 \times 600 \times \frac{7{\cdot}92-2}{7{\cdot}92} \times (22-2)} = \frac{306{,}409}{134\,545{\cdot}5} = 2{\cdot}277 \text{ sq. ins.}$$

A_c may also be found with less trouble, by knowing that the stresses are proportionate with the distance from the neutral axis; i.e., if the compression bars are the same distance from the axis as the tension bars the stress will be equal, and the same area of steel would be required. If only half the distance from the axis, the stress will be half that in the tension bars; consequently, double the quantity of steel would be required to take the same stress as taken by the tension bars. From this, the area of steel in compression must equal

the area of steel in tension multiplied by the ratio of their different distances from the axis, which is $\frac{d-n}{n-y}$,

then
$$A_c = a' \frac{d-n}{n-y}$$

Working out the steel by this equation, for comparison, we get :—

$$A_c = \frac{0{\cdot}957\,5 \times (22 - 7{\cdot}92)}{7{\cdot}92 - 2} = 2{\cdot}277, \text{ as above.}$$

For the complete beam we have the following :—

Area of steel in tension, $2{\cdot}079 + 0{\cdot}957\,5 = 3{\cdot}036$ square inches. Say 4 bars $\frac{7}{8}$ inch square.

Area of steel in compression, 2·277 square inches. Say 3 bars $\frac{7}{8}$ inch square.

Total area of steel required, 5·313,5 square inches; against 2·7 square inches in the single reinforced beam of Example VIII. An extra of 2·613,5 square inches for a saving of 2 inches in the depth of concrete.

The method used for determining the reinforcement for the beam of Example XIII. is the most satisfactory one to use when we have a beam of fixed dimensions within a reasonable limit, and when we wish to place the compressive reinforcement in the most effective position, and stress it as high as the concrete will admit, but if we design for the compressive reinforcement to be $\frac{n}{3}$ from the compressive surface we can determine the tensile and the compressive reinforcement more easily by the following more direct formulas.

(1)
$$n = \frac{m\,c\,d}{t + m\,c}$$

(2)
$$a = \frac{M}{t\left(d - \frac{n}{3}\right)}$$

(3) $$A_c = \frac{3\,(2\,t\,a - c\,b\,n)}{4\,c\,m}$$

(4) Without t and a, $$A_c = \frac{3\left[2\,M - c\,b\,n\left(d - \frac{n}{3}\right)\right]}{4\,c\,m\left(d - \frac{n}{3}\right)}.$$

If a beam is designed by these formulas the compression bars must not be placed nearer the axis than $\frac{1}{3}\,n$ from the top ; if placed at less than this from the top, which is usual, they will be stressed higher ; consequently, the beam will be stronger. It will be impossible to overstress the compression bars, as they cannot be stressed more than m times the stress in the concrete at the same depth ; therefore, as compression bars need never be placed nearer the axis than $\frac{1}{3}\,n$, these formulas can be used for designing any doubly reinforced beam ; there will, however, be a trifle more steel than there would be if the beams were designed by the formulas used for Example XIII.

Example XIV.—Determine the reinforcement, by formulas 1, 2 and 3, given at the end of the previous example, for a beam of which the full depth is 32 inches ; effective depth 30 inches ; breadth 18 inches ; span 20 feet ; load, exclusive of beam, 50,000 lb.

(1) For $c = 600$ and $t = 16\,000$, $n = \dfrac{m\,c\,d}{t + m\,c}$

$$= \frac{15 \times 600 \times 30}{16\,000 + 15 \times 600} = 10{\cdot}8 \text{ inches.}$$

Total load $= 50\,000 +$ weight of beam

$$= 50\,000 + \frac{32}{12} \times \frac{18}{12} \times 20 \times 150 = 62\,000 \text{ lb.}$$

$$M = \frac{W\ L}{8} = \frac{62\ 000 \times 20 \times 12}{8} = 1\ 860\ 000 \text{ inch-lb.}$$

$$(2)\ a = \frac{M}{t\left(d - \frac{n}{3}\right)} = \frac{1\ 860\ 000}{16\ 000\left(30 - \frac{10 \cdot 8}{3}\right)} = 4 \cdot 4 \text{ square ins.}$$

$$(3) \qquad A_c = \frac{3\ (2\ t\ a - c\ b\ n)}{4\ c\ m}$$

$$= \frac{3(2 \times 16\ 000 \times 4 \cdot 4 - 600 \times 18 \times 10 \cdot 8)}{4 \times 600 \times 15} = 2 \cdot 01 \text{ square in.}$$

This steel must not be placed further from the compression surface than $\frac{n}{3} = \frac{10 \cdot 8}{3} = 3 \cdot 6$ inches.

ECONOMIC SECTION FOR EQUAL OR UNEQUAL DOUBLE REINFORCEMENT.

Beams in which the compressive reinforcement is required to be a definite ratio of the tensile reinforcement, can be designed by formulas similar to those used for beams with single reinforcement.

Where the steel in compression is required to equal the steel in tension, i.e., when $A_c = a$, with $c = 600$, $t = 16{,}000$, and $m = 15$, the following formulas can be used.

$M = 152\ b\ d^2$ (5), p 0·010,8. (6), p being for either the compressive or tensile reinforcement only, the total proportion will be $2\ p$. $a = p\ b\ d$ (7).

Instead of using equations 6 and 8, the economic area of steel can be obtained directly by the equation,

$$a = \frac{3\ c\ b\ n}{6\ t - 4\ m\ c} \qquad (8)$$

$$\text{For 6 and 8, } n = \frac{m\,c\,d}{t + m\,c} \qquad (9).$$

c and t being given their maximum values.

For the derivations of 5, 6 and 8, see page 84 and chapter on formulas.

Example XV.—Design a beam with double and equal reinforcement for the same span and load as the beam of Example XIV.

Allowing for the weight of the beam to be the same as that of example XIV., the bending moment will be the same; then by making b a suitable proportion of d, say, $0{\cdot}6\,d$, $M = 152 \times 0{\cdot}6\,d^3$.

Hence

$$d = \sqrt[3]{\frac{M}{152 \times 0{\cdot}6}} = \sqrt[3]{\frac{1\,860\,000}{152 \times 0{\cdot}6}} = 27\tfrac{1}{2} \text{ inches.}$$

$b = 0{\cdot}6\,d = 16\frac{1}{2}$ inches, $a = p\,b\,d = 0{\cdot}010{,}8 \times 16{\cdot}5 \times 27{\cdot}5 = 4{\cdot}9$ square inches.

Checking the steel, to compare the formulas 8 and 9, with those used, we get :—

$$\text{By 9,} \qquad n = \frac{15 \times 600 \times 27{\cdot}5}{16\,000 + 15 \times 600} = 9{\cdot}9 \text{ inches.}$$

$$\text{By 8,} \quad a = \frac{3 \times 600 \times 16{\cdot}5 \times 9{\cdot}9}{6 \times 16\,000 - 4 \times 15 \times 600} = 4{\cdot}9 \text{ square inches,}$$

as before.

FOR OTHER VALUES OF c, t AND A_c

For any other values of c and t the value of M can be obtained as shown in the derivation of 5, and p, as shown for equation 6, and a as for equation 8.

If A_c is required to be some other ratio of a, say $0{\cdot}6\ a$, equation 8 becomes :—

$$a = \frac{3\ c\ b\ n}{6\ t - 0{\cdot}6 \times 4\ m\ c} \qquad (9).$$

And

$$p = \frac{n}{d} \frac{3\ c}{(6\ t - 0{\cdot}6 \times 4\ m\ c)} \qquad (10).$$

HOW TO CHECK DOUBLY REINFORCED BEAMS.

To check an existing doubly reinforced beam, the following formulas may be used, taken in the order given. Also, after designing a doubly reinforced beam by any of the methods used in the foregoing examples, it can be checked by applying these formulas ; if the results obtained do not agree with the values for which the beam was designed, it will be evidence of an error in the method, or in the calculations.

For the derivation of these formulas refer to chapter on formulas.

$$(1)\quad n = \sqrt{\frac{2\ m\ (a\ d + A_c\ y)}{b} + \left[\frac{m\ (A_c + a)}{b}\right]^2} - \frac{m\ (A_c + a)}{b}$$

If preferable in the following form.

$$n = \frac{\sqrt{2\ m\ b\ (a\ d + A_c\ y) + [m\ (A_c + a)]^2} - m\ (A_c + a)}{b}$$

$$(2)\qquad c = \frac{2\ M}{b\ n\left(d - \frac{n}{3}\right) + 2\ A_c\ m\ \frac{n - y}{n}\ (d - y)}$$

$$(3)\qquad t = c\ m\left(\frac{d - n}{n}\right)$$

$$(4)\qquad c' = c\ m\left(\frac{n - y}{n}\right)$$

Example XVI.—Checking the beam of Example XIII., where $d = 22$, $b = 14$, $A_c = 2{\cdot}277$, $a = 3{\cdot}0365$, $M = 950{,}400$.

Then $n = \sqrt{\dfrac{2m(ad + A_c y)}{b} + \left[\dfrac{m(A_c + a)}{b}\right]^2} - \dfrac{m(A_c + a)}{b} =$

$$\sqrt{\frac{2 \times 15\,(3{\cdot}0365 \times 22 + 2{\cdot}277 \times 2)}{14} + \left[\frac{15\,(2{\cdot}277 + 3{\cdot}0365)}{14}\right]^2}$$

$$- \frac{15\,(2{\cdot}277 + 3{\cdot}0365)}{14} = 13{\cdot}613 - 5{\cdot}693 = 7{\cdot}92 \text{ inches.}$$

$$c = \frac{2\,M}{b\,n\left(d - \frac{n}{3}\right) + 2\,A_c m\,\frac{n - y}{n}\,(d - y)}$$

$$= \frac{2 \times 950\;400}{14 \times 7{\cdot}92\left(22 - \frac{7{\cdot}92}{3}\right) + 2 \times 2{\cdot}27 \times 15\left(\frac{7{\cdot}92 - 2}{7{\cdot}92}\right)(22 - 2)}$$

$$= \frac{1\;900\;800}{2\;146{\cdot}63 + 1\;017{\cdot}41} = 600 \text{ lb.}$$

$$t = \frac{c\,m\,(d - n)}{n} = 15 \times 600\,\frac{22 - 7{\cdot}92}{7{\cdot}92} = 16\;000 \text{ lb.}$$

$$c' = c\,m\,\frac{n - y}{n} = 15 \times 600\,\frac{7{\cdot}92 - 2}{7{\cdot}92} = 6\;727 \text{ lb.}$$

By referring to Example XIII. the values of c, t and n will be seen to correspond exactly with those found by the above formulas.

Example XVII.—Determine the uniformly distributed load for a beam 20 feet span, 20 inches full depth, 12 inches wide, reinforced with 8 bars 1 inch square, four being 2 inches from the top, and four 2 inches from the bottom. The stresses in the concrete and steel not to exceed 600 lb. and 16,000 lb. respectively.

The bars being 2 inches from the bottom, the effective depth will be 18 inches.

The area of steel in compression, and that in tension, will be 4 square inches.

In all such cases the load must be determined from the bending moment, which must equal the moment of compression, or of tension. By using the moment of compression we have :—

$$2\,\mathrm{M} = c\left[b\,n\left(d-\frac{n}{3}\right)+2\,\mathrm{A}_c\,m\,\frac{n-y}{n}\,(d-y)\right].$$

All these factors are known except n, which can be determined by the formula,

$$n=\sqrt{\frac{2\,m\,(a\,d+\mathrm{A}_c\,y)}{b}+\left[\frac{m\,(\mathrm{A}_c+a)}{b}\right]^2}-\frac{m\,(\mathrm{A}_c+a)}{b}$$

Filling in the values, we have :—

$$n=\sqrt{\frac{2\times 15\,(4\times 18+4\times 2)}{12}+\left[\frac{15\,(4+4)}{12}\right]^2}-\frac{15\,(4+4)}{12}$$

$$=\sqrt{300}-10=7{\cdot}32 \text{ inches.}$$

Filling in the values for the moment of compression we get :—

$$M = \frac{600\left[12\times 7{\cdot}32\left(18-\frac{7{\cdot}32}{3}\right)+2\times 4\times 15\frac{7{\cdot}32-2}{7{\cdot}32}(18-2)\right]}{2}$$

$$=\frac{600\times 2760{\cdot}71}{2} = 828\ 213\cdot$$

For a distributed load, $M = \frac{W\ L}{8}$;

therefore $W = \frac{8\ M}{L} = \frac{8\times 828\ 213}{20\times 12} = 27\ 607$ lb.,

or 1,380 lb. per lineal foot.

From this we must deduct the weight of the beam, which equals $\frac{20}{12}\times 1\times 150 = 250$ lb. per lineal foot. Hence, the load for the beam will be $1380-250 = 1130$ lb. per lineal foot.

The tension in the steel can be determined by the equation,

$$t = \frac{c\,m\,(d-n)}{n} = \frac{600\times 15\ (18-7{\cdot}32)}{7{\cdot}32} = 13\ 131 \text{ lb.}$$

The compression in the steel, by the equation

$$c' = c\,m\,\frac{n-y}{n} = \frac{600\times 15\ (7{\cdot}32-2)}{7{\cdot}32} = 6\ 541 \text{ lb.}$$

The steel is thus shown to be much understressed, but the load is a maximum for the beam, for if it was increased the concrete would be overstressed.

Further examples of beams are given with various structures in Part II.

SLABS AND FLOORS.

There are several kinds of concrete floors, but which to use depends upon the class of building, and the comparative costs and advantages of the different kinds. The most simple, and generally the cheapest, if the spans

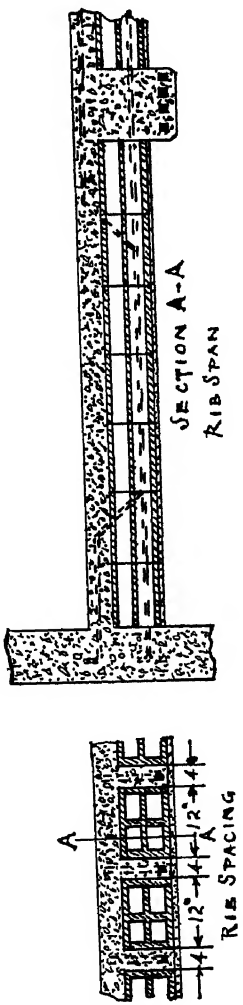


FIG. 70.

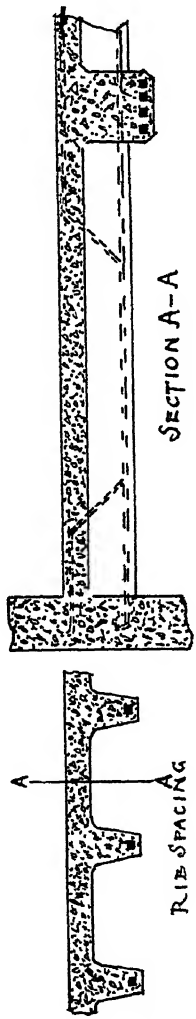


FIG. 71.

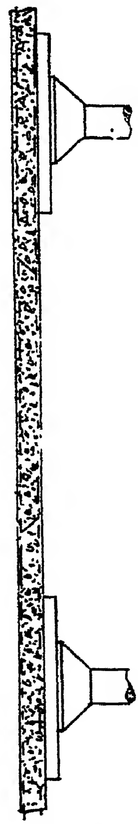


FIG. 72.

are not too great, is the plain flat slab and beam system ; in many buildings, however, it is desirable to keep the ceilings free of projecting beams ; in such cases, beamless systems are required ; but in any case, where beams are not objected to, it is usual to plan the floors so that as many as possible of the beams will come over partition walls. With large spans between the beams the floor slabs generally work out to an excessive thickness, and are, therefore, heavy and costly. In such cases a lighter form of construction is desirable, and is often obtained by adopting either a rib and tile slab, as Fig. 70, or what is known as a rib, or concrete joist, slab, as Fig. 71. Another system largely used, particularly for industrial buildings, warehouses, and public garages, or where it is required to avoid the loss of space in the height of the rooms, occasioned by projecting beams, is the flat slab system,. as Fig. 72.

The strength of slabs is determined in a similar manner to that of beams, and if they are supported, or fixed, at two sides only, the bending moment will be the same as for a beam similarly supported or fixed, but with a slab supported, or fixed, at all four sides, the bending moment will not be so great as that for a beam of the same span, owing to the slab being strengthened by the supports at the ends as well as at the sides. Authorities do not agree as to the exact difference, which depends upon the proportion of the load carried by the respective supports.

A square slab supported all round is twice as strong as the same slab supported at two sides only ; this ratio, however, diminishes as the length increases, until the length exceeds a trifle more than twice its width, then the proportion of the load borne by the end supports can be ignored, as the effect on the strength is too slight to be considered ; consequently, the slab may be taken as a simple beam, of length equal to the width of the slab.

Fig. 73 illustrates how a rectangular plain concrete slab, supported all round, generally fractures when overloaded. The end supports are considered to carry

the load on the triangular pieces A and B, and the side supports that on the pieces C and D. In a square slab the fractures take place along the diagonals of the slab, thus making an angle of 45 degrees with the ends, but as the slab increases in length, the lines of fracture make a less angle with the ends; consequently, with a square slab the supports are equally loaded, and as the length increases the load increases on the side supports and decreases on the end supports.

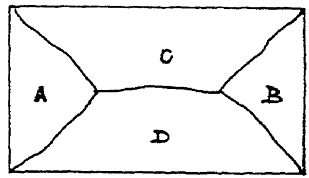


FIG. 73.

It is therefore obvious that the reinforcement must be designed for the width, and for the length, of slabs that are less in length than twice their width. This can be done, as with beams, by equating the bending moment with the moment of resistance. The proportion of the load borne by the opposite supports must be determined for the bending moment, or the bending moment must be obtained for the full load and proportionately reduced for the width and length.

For this purpose the following equations may be used, which will give the proportions in accordance with the foregoing explanation :

$$f = \frac{0 \cdot 05\ B + 0 \cdot 45\ L}{B} \qquad r = \frac{0 \cdot 95\ B - 0 \cdot 45\ L}{B}$$

Or $r = 1 - f$, as $f + r = 1$. Where B = breadth of the slab.
L = length of the slab.

f = proportion of the load borne by the side supports, or the factor to reduce the bending moment when the slab is taken as a beam of length equal to the breadth of the slab.

r = proportion of the load borne by the end supports, or the factor to reduce the bending moment when the slab is taken as a beam of length equal to the length of the slab.

Other rules are used for the purpose, but no two exactly agree, the most general being the two given below; the derivations of these are not very clear, but the rules are given, with the author's, for comparison, and use if preferred.

When $\frac{L}{B}=$	Grashof and Rankine		French Government	
	$f=\frac{L^4}{L^4+B^4}$	$r=\frac{B^4}{L^4+B^4}$	$f=\frac{1}{1+\frac{2B^4}{L^4}}$	$r=\frac{1}{1+\frac{2L^4}{B^4}}$
1·0	0·50	0·50	0·33	0·33
1·25	0·71	0·29	0·55	0·17
1·5	0·83	0·16	0·71	0·09
1·75	0·90	0·07	0·83	0·05
2·0	0 94	0·05	0·89	0·03

By the Author.

$\frac{L}{B}=$	$f=\frac{0\cdot05\text{ B}+0\cdot45\text{ L}}{\text{B}}$	$r=\frac{0\cdot95\text{ B}-0\cdot45\text{ L}}{\text{B}}$
1·0	0·50	0·50
1·25	0·613	0 387
1·5	0·725	0·275
1·75	0·837	0 163
2·0	0·95	0·05

Example XVIII.—Determine the safe load for a granite concrete slab, 10 feet by 8 feet, 5 inches thick, supported all round and reinforced in both directions with ½ inch round bars placed 6 inches apart, center to center. The bars across the slab are placed on top of the longitudinal bars, the axis of the latter being 1 inch from the bottom of the slab.

Taking the breadth first, the bending moment will be :—

$$\frac{\text{W L}}{8}f\text{, and } f=\frac{0\cdot05\text{ B}+0\cdot45\text{ L}}{\text{B}}=\frac{0\cdot05\times8+0\cdot45\times10}{8}=0\cdot613.$$

Hence $$M = \frac{W\,8 \times 12 \times 0{\cdot}613}{8}.$$

To determine W we must find the moment of resistance, which, for compression, will equal

$$\frac{c\,b\,n}{2}\left(d - \frac{n}{3}\right).$$

Taking one lineal foot of slab, b will be 12 inches, $a = 2 \times 0{\cdot}1963 = 0{\cdot}392\ 6$ square inch, and as these bars are on top of the longitudinal bars, d will be 3·5 inches.

Then $$n = \sqrt{\frac{2\,m\,a\,d}{b} + \left(\frac{a\,m}{b}\right)^2} - \frac{a\,m}{b} =$$

$$\sqrt{\frac{2 \times 0{\cdot}392\ 6 \times 15 \times 3{\cdot}5}{12} + \left(\frac{0{\cdot}392\ 6 \times 15}{12}\right)^2} - \frac{0{\cdot}392\,6 \times 15}{12}$$

$$= \sqrt{3{\cdot}665} - 0{\cdot}49 = 1{\cdot}43 \text{ inches.}$$

Hence, the moment of resistance equals

$$\frac{600 \times 12 \times 1{\cdot}43\left(3{\cdot}5 - \frac{1{\cdot}43}{3}\right)}{2} = 15\ 567 \text{ inch-lb.}$$

Equating this with the bending moment we have :—

$$\frac{8\,W \times 12 \times 0{\cdot}613}{8} = 15\ 567.$$

Therefore $$W = \frac{15\ 567}{12 \times 0{\cdot}613} = 2\ 116 \text{ lb.}$$

This load is for a portion of the slab 1 foot by 8 feet ;

therefore, the load per square foot, including the weight of the slab, will be $\frac{2\,116}{8} = 264{\cdot}5$ lb. The weight of the slab per square foot equals $150 \times \frac{5}{12} = 62{\cdot}5$ lb.; deducting this we get $264{\cdot}5 - 62{\cdot}5 = 202$ lb. per square foot for the external load.

The stress in the steel equals

$$t = c\,m\,\frac{d-n}{n} = \frac{600 \times 15\,(3{\cdot}5 - 1{\cdot}43)}{1{\cdot}43} = 13\,028 \text{ lb.},$$

which is well within the limit.

Checking the reinforcement along the length of the slab, or the stress in the opposite direction to those already obtained,

$$M = \frac{W\,Lr}{8}, \text{ and } r = \frac{0{\cdot}95\,B - 0{\cdot}45\,L}{B} = 0{\cdot}387.$$

The load for the 12 inch strip equals $264{\cdot}5 \times 10 = 2\,645$ lb.;

$$\text{then } M = \frac{2\,645 \times 10 \times 12 \times 0{\cdot}387}{8} = 15\,354 \text{ lb.};$$

$$\text{therefore } 15\,354 = \frac{c\,b\,n\left(d - \frac{n}{3}\right)}{2}, \quad d = 4 \text{ inches},$$

and

$$n = \sqrt{\frac{2 \times 0{\cdot}392\,6 \times 15 \times 4}{12} + \left(\frac{0{\cdot}392\,6 \times 15}{12}\right)^2}$$

$$- \frac{0{\cdot}392\,6 \times 15}{12} = 1{\cdot}55 \text{ inch};$$

$$\text{hence, } c = \frac{2 \times 15\,341}{b\, n\left(d - \left(\frac{n}{3}\right)\right)} = \frac{30\,682}{12 \times 1{\cdot}55\left(4 - \frac{1{\cdot}55}{3}\right)} = 474 \text{ lb.}$$

$$t = c\, m\, \frac{d-n}{n} = \frac{474 \times 15\ (4 - 1{\cdot}55)}{1{\cdot}55} = 11\,238 \text{ lb.}$$

With an external load of 202 lb. per square foot we get the following :—The stress in the concrete, acting across the slab, equals 600 lb. per square inch. The stress in the steel acting across the slab equals 13 028 lb. per square inch. The stress in the concrete acting along the slab equals 474 lb. per square inch. The stress in the steel acting along the slab equals 11 238 lb. per square inch.

Example XIX.—Design a floor for a span of 8 feet by 12 feet, to carry a load of 160 lb. per square foot, exclusive of its own weight.

Using hard stone concrete ; thus allowing for c to be 600, and t to be 16 000, and m to be 15.

Taking the breadth first,

$$M = \frac{W\ L\ f}{8},$$

$$f = \frac{0{\cdot}05\ B + 0{\cdot}45\ L}{B} = \frac{0{\cdot}05 \times 8 + 0{\cdot}45 \times 12}{8} = 0{\cdot}725.$$

W equals the external load plus the weight of the slab. Taking a 12 inch strip across the breadth, and assuming the full depth will be about 5 inches, W will be $(160 + 150 \times \frac{5}{12})\ 8 = 222{\cdot}5 \times 8 = 1\ 780$ lb.

$$\text{Then} \quad M = \frac{1\,780 \times 8 \times 12 \times 0{\cdot}725}{8} = 15\,486 \text{ lb.}$$

For the economic section use the same equations as for beams,

$$\text{where} \quad d = \sqrt{\frac{M}{95\,b}} = \sqrt{\frac{15\ 486}{12 \times 95}} = 3{\cdot}7 \text{ inches.}$$

Area of steel required equals $p\ b\ d$, and p, for these stresses, equals 0·006 75, see page 85; therefore a = 0·006 $75 \times 12 \times 3{\cdot}7$ = 0·3 square inch, say bars $\frac{7}{16}$-inch square placed 6 inches apart.

For the reinforcement along the length of the slab, taking a 12 inch strip, W = $222{\cdot}5 \times 12$ = 2 670 lb.

$$M = \frac{W\ L\ r}{8} = \frac{2\ 670 \times 12 \times 12 \times 0{\cdot}275}{8} = 13\ 216{\cdot}5 \text{ lb.}$$

The depth of the reinforcement, to give the same stresses, will be :—

$$\sqrt{\frac{M}{95\,b}} = \sqrt{\frac{13\ 216{\cdot}5}{12 \times 95}} = 3{\cdot}4 \text{ inches.}$$

$a = p\,b\,d = 0{\cdot}006\ 75 \times 12 \times 3{\cdot}4 = 0{\cdot}276$ square inch. Say 2 bars $\frac{3}{8}$ inch square, per foot of width.

The difference in the required depth for the bars is 3·7 — 3·4, only 0·3 inch, and the distance between the axis of the bars is 0·4 inch; consequently, the cross bars must be placed 3·8, say 4 inches from the top. With 1 inch cover, the full depth will be 5 inches, the same as allowed for in estimating the load. If $\frac{7}{16}$-inch bars are also used for the top, instead of $\frac{3}{8}$ inch, the depth need not be altered.

To avoid errors in construction it is advisable to use the same size rods throughout and to effect economy in the spacing, rather than use rods almost equal in size with equal spacing. The spacing for any size rods can be determined by the formula used to determine the area, thus :—$a = p\ b\ d$; therefore, the breadth, or distance

apart for rods for any area will equal $b = \frac{a}{p\ d}$. In the present example, if we use $\frac{7}{16}$-inch square rods along the length of the slab their distance apart will equal

$$\frac{\frac{7}{16}}{0{\cdot}006\ 75 \times 3{\cdot}4} = 8{\cdot}3 \text{ inches.}$$

Example XX.—This example is taken for the purpose of comparing the strength of a coke-breeze concrete slab with a granite, or hard stone concrete slab, and to introduce the different values required for designing.

Determine the thickness, and the reinforcement, for a coke-breeze floor slab for the same span and load as that of Example XIX.

The weight of coke-breeze concrete can be taken as 130 lb. per cubic foot.

Allowing for a thickness of 6 inches, the load per square foot will be $160 + \frac{130}{2} = 225$ lb.

Taking a 12 inch strip across the slab,

$$M = \frac{W\ L\ f}{8} = \frac{225 \times 8 \times 8 \times 12 \times 0{\cdot}725}{8} = 15\ 660 \text{ inch-lb.}$$

For the economic section, for coke-breeze, where

$c = 250$, $t = 16\ 000$, and $m = 30$, $d = \sqrt{\frac{M}{36b}}$

$$= \sqrt{\frac{15\ 660}{36 \times 12}} = \sqrt{36{\cdot}25} = 6{\cdot}02.$$

The area of steel can be determined by the equation $a = p\ b\ d$, where $p = \dfrac{c\ n}{2\ t\ d}$, or by equating compression and tension, thus :—

$$t\ a = \frac{c\ b\ n}{2}\ ;\ \text{therefore}\ \frac{c\ b\ n}{2\ t} = a,$$

and

$$n = \frac{m\ c\ d}{t + m\ c} = \frac{30 \times 250 \times 6 \cdot 02}{16\ 000 + 300 \times 250} = \frac{45\ 150}{23\ 500} = 1 \cdot 92 \text{ inch ;}$$

$$\text{therefore } a = \frac{250 \times 12 \times 1 \cdot 92}{2 \times 16\ 000} = 0 \cdot 18 \text{ square inch.}$$

Using square bars, the most suitable size will be $\frac{5}{16}$-inch; two of these per foot will give 0·196 square inch, or the distance apart for the rods to give the correct amount will equal $b = \dfrac{a}{p\ d}$, but as we have not got p already worked out for these stresses, it will be quicker to determine b by proportion, thus :—

$$0 \cdot 18 : 12 \text{ inches} :: \left(\frac{5}{16}\right)^2 : b\ ;$$

$$\text{therefore} \qquad b = \frac{12 \times (\frac{5}{16})^2}{0 \cdot 18} = 6 \cdot 5 \text{ inches.}$$

For the reinforcement along the length of the slab, taking a 12 inch strip,

$$M = \frac{W\ L\ r}{8} = \frac{225 \times 12 \times 12 \times 12 \times 0 \cdot 275}{8} = 13\ 365 \text{ inch-lb.}$$

$$d = \sqrt{\frac{M}{36\ b}} = \sqrt{\frac{13\ 365}{36 \times 12}} = 5 \cdot 56 \text{ inches,}$$

and $$n = \frac{30 \times 250 \times 5\cdot56}{16\,000 + 30 \times 250} = 1\cdot77 \text{ inch};$$

then $$a = \frac{c\,b\,n}{2\,t} = \frac{250 \times 12 \times 1\cdot77}{2 \times 16\,000} = 0\cdot166 \text{ square inch.}$$

The same size rods can be used as for across the slab.

As the depth to the bottom of the rods must be 6 inches, by allowing 1 inch cover, the full thickness will be 7 inches, 1 inch more than allowed for in estimating the load ; this, however, is not enough to make any appreciable difference in the stresses.

Comparing the two cases we have :—

	Thickness of Slab.	Area of Long Bars.	Area of Cross Bars	Total Area.
Coke-breeze Concrete	7 inches	0·166 square inch per ft	0·18 square inch per ft.	0 346 inch per square foot
Stone Concrete	5 inches	0·276 square inch per ft	0·3 square inch per ft	0 576 inch per square foot

Slabs securely fixed in the walls, or continued over beams or other supports that are perfectly level and rigid, must be considered the same as fixed beams ; they are $1\frac{1}{2}$ times as strong as slabs merely supported. They can be designed in a similar manner to those already described, but there will be a reverse tensional stress over the supports, where the bending moment will be $\frac{W\ L}{12}$, and the bending moment at the center of the span be $\frac{W\ L}{24}$. To guard against imperfect fixing, or deflection of the supports, it is, however, advisable to take the bending moment for both the center and the ends as $\frac{W\ L}{12}$. The same area of reinforcement will then be

required for both positions, and it can be arranged to meet the reverse stress by bending it up to pass the center of the depth at one-fourth the span, as Fig. 74.

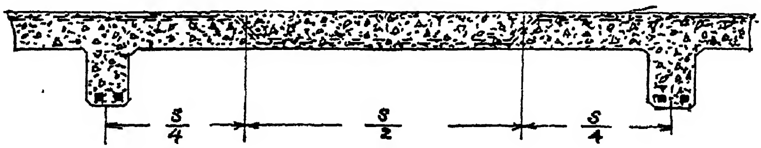


FIG. 74.

RIB AND TILE FLOORS.

The object of this class of floor is to keep the ceilings level and as free as possible from beams. To attain this object the floor slabs are designed to span the full width and length of the rooms, except in exceptionally large spans ; consequently, they often turn out to an excessive thickness and weight. To keep the weight at a minimum, all useless concrete is removed and replaced with light hollow clay tile.

In designing a slab of this class we have to consider how much of the concrete is useless, or how much can be removed without affecting the combined resistance of the concrete and steel. For this we must bear in mind the fact that the steel must be encased with sufficient concrete to protect it from fire and water, and sufficient to provide for the bond and shear stresses. The concrete above the neutral axis is designed to take the direct compression ; consequently, none of this can be removed ; that below the axis is in the tension area, but is not designed to take tension ; it is, however, resisting shear, and at least half the shear should be taken by the concrete ; therefore, if sufficient is retained for this purpose, the remainder can be removed, as in the following example.

Example XXI.—Design a rib and tile floor slab for an effective span of 20 feet. The load to be carried, exclusive of its own weight, to be 100 lb. per square foot.

Allowing for the weight of slab to be 100 lb. per square foot, the total load for 1 foot by 20 feet of slab will be $200 \times 20 = 4{,}000$ lb.

Taking the slab as simply supported,

$$M = \frac{W\ L}{8} = \frac{4\,000 \times 20 \times 12}{8} = 120\,000 \text{ inch-lb.}$$

By the usual formula :—

$$d = \sqrt{\frac{120\,000}{95 \times 12}} = 10\tfrac{1}{2} \text{ inches,}$$

or a full depth of $11\frac{1}{2}$ inches.
$a = p\ b\ d = 0{\cdot}006\,75 \times 12 \times 10{\cdot}25 = 0{\cdot}83$ square inch.

The size rods to use will depend on the spacing, which will be governed by the size of the tile ; those generally used are 12 inches wide, which is a standard size. Using this size and allowing for a 4 inch rib, the spacing for the rods will be 16 inches ; then $a = p\ b\ d = 0{\cdot}006\,75 \times 16 \times 10{\cdot}25 = 1{\cdot}107$ square inch ; which will be provided by 1 $\frac{3}{16}$-inch round bar, or 1 $\frac{1}{16}$-inch square bar. There should in no case be less than a 3-inch rib to encase the rods. The standard size tile should be considered together with the thickness of the ribs and spacing of the rods.

The thickness of the slab above the tile must also be considered with the thickness of the standard tile ; in no case should there be less concrete than the depth to the neutral axis, which thickness, for this case, where the concrete and steel stresses are 600 and 16 000 lb. respectively, will be $0{\cdot}36\ d = 0{\cdot}36 \times 10{\cdot}25 = 3{\cdot}69$ inches. The concrete can, therefore, be removed to within, say, $3\frac{3}{4}$ inches of the top. This would leave $7\frac{3}{4}$ inches for the

tile, of which the standard thickness is 8 inches ; therefore, the full depth of the concrete should be not less than $11\frac{3}{4}$ inches. The section will be as Fig. 75.

The thickness of the slab should never be less than 3 inches, and if less than the depth to the neutral axis, each rib should be designed separately as a beam ; for which see chapter on Tee-Beams.

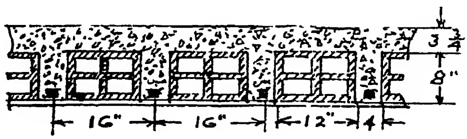


FIG. 75.

The weight of the slab should now be computed, and, if necessary, adjustments made accordingly.

It will be noticed that the reinforcement must be designed for one way only, which should be across the short span. The shear and bond stresses must be considered as explained for tee beams.

In checking floor slabs of this description they must be taken as a series of tee beams, the width of the table being taken from the center of the tile on each side of the rib ; see chapter on Tee Beams. To check, determine the neutral axis first ; if this turns out above the bottom of slab, complete the checking by the formulas for rectangular beams. If the axis is below the slab, check by the tee beam formulas.

Instead of filling in with hollow tile, the space can be left and metal lath and plaster attached to the bottom of the ribs by means of wire hangers being embedded in the concrete.

RIB SLABS.

Rib slabs are used for the same reason as rib and tile slabs, but where it is not necessary to have the ceilings level. The cost of these is a little less than rib and tile.

when standard steel forms are used, but if steel or wood forms have to be specially made for the job there is not likely to be any appreciable difference in the cost of the two systems.

Rib slabs are designed in the same way as the rib and tile, in the previous example, excepting that the clear space between the ribs is generally taken between 20 and 26 inches. The ribs have sloping sides caused by the forms being tapered to facilitate removal. The sloping sides are also an advantage in increasing the shear area of the concrete.

Example XXII.—Determine the thickness of slab, size of ribs and the reinforcement for a rib slab to carry a load of 100 lb. per square foot. The span being 24 feet, and the bending moment $\frac{W\ L}{10}$. The distance between the bottom of the ribs to be 24 inches.

Taking a portion of the slab 1 foot wide, 24 feet long, and allowing for its weight to be 100 lb. per square foot, the total load for this portion will be $(100+100)\ 24 =$ 4 800 lb.

Then $$M = \frac{4\ 800 \times 24 \times 12}{10} = 138\ 240 \text{ inch-lb.}$$

$$d = \sqrt{\frac{M}{95\ b}} = \sqrt{\frac{138\ 240}{95 \times 12}} = 11 \text{ inches.}$$

$a = p\ b\ d$, $p = 0{\cdot}006\ 75$, $d = 11$, $b =$ distance apart of the bars, which, by allowing for a 4 inch rib, will be 28 inches. Then $a = 0{\cdot}006\ 75 \times 11 \times 28 = 2{\cdot}079$ square inches. Two bars 1 inch square will provide 2 square inches. Two bars 1 $\frac{1}{16}$-inch square will provide 2·257 8 square inches. In practice the 1 inch bars would be considered sufficient.

Depth of the neutral axis, for the thickness of the slab, will equal $0 \cdot 36\, d = 3 \cdot 96$, say 4 inches. Allowing $1\frac{1}{2}$ inch for cover to the steel, the section will be as Fig. 76.

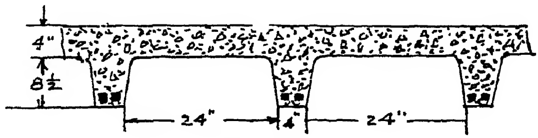


FIG. 76.

Further particulars on slabs are given in the examples dealing with floors.

TEE-BEAM FLOORS.

When constructing a floor consisting of concrete beams and slabs, the concrete for the beam and slab above the beam, must be filled in, to the top of the slab in one operation, to ensure perfect cohesion between the slab and beam. The thickness of the slab can then be included in the depth of the beam, and a portion of the slab can be added to the breadth of the beam, thus considerably increasing the compression area. No definite rule can be laid down to determine the precise width of the slab acting with the beam, for this is influenced by the thickness and width of the slab between the beams, the position of the reinforcement, amount of load, and with the slightest variation in the quality of the concrete. It is therefore, advisable for the designer to use his own judgment as to the width to allow for.

When the slabs require reinforcing in one direction only, and that at right angles to the tee beams, the compression in the slab or table of the beam, acting with the beam, will be in the opposite direction to that acting with the slab and will therefore not affect the compression as usually taken in the slab ; consequently, a greater width of slab can be assumed as acting with the beam than if the slab requires reinforcing in both directions, but in no case should an amount be assumed of more than

three-eighths of the slab on either side, making the maximum width of the table equal to three fourths of the distance between the centers of the beams ; nor must it be more than 15 times the thickness of the slab, or one-third the span of the tee beam, or six times the width of the rib of the tee beam, whichever is the least ; no part of the slab will then be taken as acting with more than one beam. When the slabs are reinforced in both directions, the total width of the table should in no case be taken as more than half the distance between the centers of the beams, or the overhanging width on each side of the web not more than six times the thickness of the slab, or the full width of the table not more than one quarter the span of the beam. When secondary beams are used at right angles to the main beams, the secondary beams must be considered as the tee beams, and the main beams designed as rectangular beams supporting the secondary beams and slabs. Without secondary beams, the main beams will be tee beams.

If the neutral axis coincides with, or is above, the bottom of the slab, all the equations for singly reinforced rectangular beams can be used for designing or checking existing beams, but B must be substituted for b, where B equals the width of the table acting with the beam, as Fig. 77.

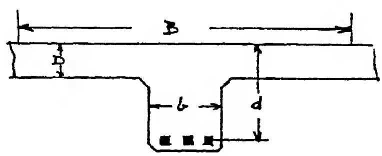


FIG. 77.

If the axis is below the slab, the whole thickness of the slab and a portion of the leg of the beam will be in compression.

The compression in the slab will be

$$\frac{\mathrm{BD}\left(c+c\,\dfrac{n-\mathrm{D}}{n}\right)}{2}$$

The compression in the leg of the beam will be

$$\frac{b\,c\,\dfrac{(n-\mathrm{D})^2}{n}}{2}$$

The total compression will thus equal one-half of

$$\mathrm{B\,D}\left(c + c\,\frac{n-\mathrm{D}}{n}\right) + \frac{b\,c\,(n-\mathrm{D})^2}{n}$$

Where D = thickness of the slab, and b = the thickness of the leg.

With these conditions the center of compression will be less than $\frac{n}{3}$ from the surface owing to the loss of compression between the bottom of the slab and the neutral axis.

If we neglect the compression in the leg—which can be done in small beams as it is too slight to make any appreciable difference—the center of compression can be obtained by the equation

$$g = \frac{3\,\mathrm{D}\,n - 2\,\mathrm{D}^2}{6\,n - 3\,\mathrm{D}},$$

where g = the distance of the center of compression from the compression surface.

In large beams, however, the compression in the leg should be considered, especially with a high percentage of reinforcement when the axis may be a considerable distance below the slab; then

$$g = \frac{\mathrm{B\,D}^2\,(3\,n - 2\,\mathrm{D}) + b\,(2\,\mathrm{D} + n)\,(n-\mathrm{D})^2}{3\,[\mathrm{B\,D}\,(2\,n - \mathrm{D}) + b\,(n - \mathrm{D})^2]}$$

The neutral axis, t and c, can be determined by the following formulas :—

$$(1) \quad n = \sqrt{\frac{2\,m\,a\,d + D^2 (B-b)}{b} + \left(\frac{D\,(B-b) + m\,a}{b}\right)^2} - \frac{D\,(B-b) + m\,a}{b}.$$

$$(2) \qquad t = \frac{M}{a\,(d-g)}.$$

$$(3) \qquad c = \frac{2\,M\,n}{(d-g)\,[B\,D\,(2\,n - D) + b\,(n-D)^2]}.$$

If t is determined by 2,

$$(4) \qquad c = \frac{t\,n}{m\,(d-n)}.$$

Example XXIII.—Determine the load for one of a series of beams 20 feet effective span, placed at 8 feet centers, and supporting a floor slab 6 inches thick, reinforced at right angles to the beams. The beams are 18 inches deep from the top of the slab to the center of the reinforcement, and 12 inches wide. The reinforcement in each beam consists of 4 bars $1\frac{1}{4}$ inch square. There are no secondary beams.

Taking the breadth of the slab acting with the beam in accordance with the rules on page 140, B will be 72 inches.

The load must be determined by equating the bending moment with the moment of resistance of the concrete, or of the steel, whichever is least when the maximum values are given to c and t.

The formulas to use for c and t will depend upon the position of the neutral axis; if this is below the slab we must use the formulas for tee beams; if it is within the thickness of the slab the more simple formulas for

rectangular beams will apply Trying the latter for the neutral axis we get :—

$$n = \sqrt{\frac{2\,m\,a\,d}{\mathrm{B}} + \left[\frac{m\,a}{\mathrm{B}}\right]^2} - \frac{m\,a}{\mathrm{B}} =$$

$$\sqrt{\frac{2 \times 15 \times 6{\cdot}25 \times 18}{72} + \left(\frac{15 \times 6{\cdot}25}{72}\right)^2} - \frac{15 \times 6{\cdot}25}{72} = 5{\cdot}667$$

which is less than the thickness of the slab, therefore all the formulas for rectangular beams will apply. Hence, the moment of tension equals

$$t\,a\left(d - \frac{n}{3}\right) = 16\,000 \times 6{\cdot}25\left(18 - \frac{5{\cdot}66}{3}\right) = 1\,611\,100 \text{ inch-lb.}$$

Before taking this to find the load, see if the concrete would be overstressed with 16 000 lbs. in the steel, for, if so, it will be evident that the bending moment must be obtained by the moment of compression.

By No. 4, page 143,

$$c = \frac{t\,n}{m\,(d - n)} = \frac{16\ 000 \times 5{\cdot}66}{15(18 - 5{\cdot}66)} = 489 \text{ lb.}$$

By this we see that the load must be determined by the moment of tension, for if we use the equation

$$\mathrm{M} = \frac{c\,\mathrm{B}\,n\left(d - \frac{n}{3}\right)}{2}$$

and insert the value of 600 for c, the bending moment would be much more than that already found, and the steel would be overstressed. Therefore, stressing the steel to 16 000 lbs. per square inch, there will be only 489 lb. per square inch compression in the concrete, and the bending moment will be 1,611,100 lb.

Taking the beams as simply supported at the ends,

$$M = \frac{W\ L}{8},$$

therefore

$$W = \frac{8\ M}{L} = \frac{8 \times 1\ 611\ 100}{20 \times 12} = 53\ 703 \text{ lb.}$$

To find the live load for the floor, the weight of the floor itself must be deducted from the 53 703 lb. The portion of slab supported by one beam is equal to one-half the slab on each side; therefore the weight equals $8 \times 20 \times \frac{6}{12} \times 150$ lb. = 12 000 lb.

Weight of leg of beam = $1 \times 1 \times 20 \times 150$ = 3 000 lb.

Total weight of each bay of floor = 15 000 lb.

Safe load for floor = 53 703 — 15 000 lb. = 38 703 lb. Say 240 lb. per square foot.

Example XXIV.—Determine the load for one of a series of beams 24 feet span, placed at eight feet centers, supporting a floor slab 4 inches thick reinforced at right angles to the beams, which have a full depth of 26 inches, breadth of 12 inches, and reinforcement consisting of 5 rods $1\frac{1}{4}$ inch square.

As in the case of Example XXIII., the load must be determined by the moment of tension or compression, but in this case the slab being thin in relation to the depth of the beam the neutral axis will be below the bottom of the slab; consequently the formulas for rectangular beams will not apply; we must therefore use the formulas for tee beams.

Taking the beams as simply supported at their ends,

$M = \frac{W\ L}{8}$ which must equal the moment of tension,

$= t\ a\ (d - g).$

L

To determine g we must obtain the neutral axis which equals

$$\sqrt{\frac{2\,m\,a\,d + D^2\,(B-b)}{b} + \left[\frac{m\,a + D\,(B-b)}{b}\right]^2} - \frac{m\,a + D\,(B-b)}{b}.$$

By rules on page. 141, B, the breadth of slab to be assumed as acting with, the beam, is governed by the thickness of the slab, and must not exceed 60 inches.

Hence

$$n = \sqrt{\frac{2\times 15\times 7\cdot 8\times 24 + 4^2(60-12)}{12} + \left[\frac{15\times 7\cdot 8 + 4(60-12)}{12}\right]^2} - \frac{15\times 7\cdot 8 + 4\,(60-12)}{12}$$

$$= 34\cdot 57 - 25\cdot 75 = 8\cdot 82.$$

If we neglect the compression in the leg of the beam

$$g = \frac{3\,D\,n - 2\,D^2}{6\,n - 3\,D} = \frac{3\times 4\times 8\cdot 8 - 2\times 16}{6\times 8\cdot 8 - 3\times 4} = 1\cdot 8.$$

But to be correct the compression on the leg should be considered, then

$$g = \frac{B\,D^2\,(3\,n - 2\,D) + b\,(2\,D + n)\,(n - D)^2}{3\,[B\,D\,(2\,n - D) + b\,(n - D)^2]}$$

$$= \frac{60\times 4^2(3\times 8\cdot 8 - 2\times 4) + 12\,(2\times 4 + 8\cdot 8)\,(8\cdot 8 - 4)^2}{3\,[30\times 4\,(2\times 8\cdot 8 - 4) + 12\,(8\cdot 8 - 4)^2]} = 2\cdot 1.$$

The moment of tension,

$$t\,a\,(d - g) = 16\,000\times 7\cdot 8\,(24 - 2\cdot 1) = 2\,737\,500.$$

With this moment and 16 000 lb. in the steel the stress in the concrete will equal

$$c = \frac{t\,n}{m\,(d - n)} = \frac{16\,000 \times 8{\cdot}8}{15\,(24 - 8{\cdot}8)} = 620 \text{ lb.}$$

which is satisfactory.

If c turned out more than allowed, it would be necessary for us to determine the bending moment by the moment of compression, by the formula, page 143, instead of by the moment of tension.

$$M = \frac{W\,L}{8} = 2\,737\,500 \text{ inch-lb.}$$

Hence $$W = \frac{8 \times 2\,737\,500}{24 \times 12} = 76\,042 \text{ lb.}$$

To determine the live load for the beam the weight of the floor slab and beam must be deducted from the 76 042 lb. The portion of slab supported by one beam is equal to one-half the slab on each side ; therefore the weight equals $24 \times 8 \times \frac{4}{12} \times 150 = 9\,600$ lb. The weight of the leg of the beam equals $\frac{22}{12} \times 1 \times 24 \times 150 =$ 6 600 lb. Weight of slab and beam = 16 200 lb. The live load for the beam equals 76 042 — 16 200 = 59 842 lb.

Example XXV.—Design a floor for a room 60 feet × 20 feet, to support a safe load of 150 lb. per square foot.

The floor may be divided into slabs 20 feet × 10 feet by placing beams across the room at 10 feet centers, as the length of each slab will then be twice its breadth, each may be taken as a beam 20 feet × 10 feet, the reinforcement being designed for across the 10 feet span only. Taking one lineal foot of the length of the slab, we then have a beam 12 inches wide by 10 feet long.

The live load will be $150 \times 10 = 1\,500$ lb. To this must be added the weight of the slab itself; assuming this to work out to about 6 inches thick, its weight will be $10 \times \frac{6}{12} \times 150 = 750$ lb. Hence, $W = 2\,250$ lb. Being continuous slabs supported by beams, the bending moment can be taken as

$$\frac{W\,L}{12} = \frac{2\,250 \times 10 \times 12}{12} = 22\,500 \text{ inch-lb.}$$

For the economic section,

$$d = \sqrt{\frac{M}{95\,b}} = \sqrt{\frac{22\,500}{95 \times 12}}, \text{ say } 4\tfrac{1}{2} \text{ inches.}$$

As not less than 1 inch must be allowed for cover to the bars, the full depth may be taken as $5\frac{1}{2}$ inches.

$a = p\ b\ d = 0{\cdot}006\,75 \times 12 \times 4\frac{1}{2} = 0{\cdot}365$ square inch, say 2 bars $\frac{7}{16}$ inch square per lineal foot of slab, or $\frac{1}{2}$ inch bars at 8 inch centers.

The beams may be taken as tee beams of breadth not more than one-third of the span, say 80 inches, or not greater than three-quarters the width of the slab. The former, being the smaller, is the amount to use.

The load for each will be the live load and the weight of the slab for one bay, plus the weight of the portion of beam projecting below the slab.

The load for one slab equals $2\,250 \times 20 = 45\,000$ lb. Allowing for the leg to be about 12 inches square, its weight will be $150 \times 20 = 3\,000$ lb. Therefore $W = 3\,000 + 45\,000 = 48\,000$ lb.

$$M = \frac{W\,L}{8} = \frac{48\,000 \times 20 \times 12}{8} = 1\,440\,000 \text{ inch-lb.}$$

By the formulas for beams, but substituting B for b,

$$d = \sqrt{\frac{M}{95\,B}} = \sqrt{\frac{1\,440\,000}{95 \times 80}} = 13{\cdot}76 \text{ inches.}$$

$a = p\,B\,d = 0{\cdot}006\,75 \times 80 \times 13{\cdot}76 = 7{\cdot}43$ square inches, say 5 bars $1\frac{1}{4}$ inch square.

The width of the leg should not be less than one-sixth of the width taken for the table of the beam, say 13 inches. The longitudinal and transverse sections of the floor will be as Figs. 78 and 79.

The shearing stress is considered further on.

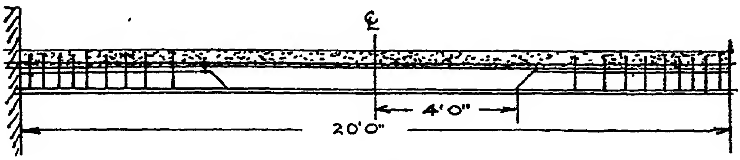


FIG. 78.

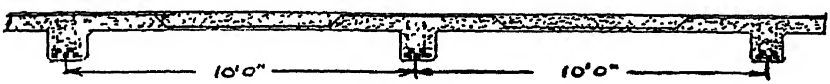


FIG. 79.

Example XXVI.—Design a warehouse floor of the slab and beam type, 60 feet long and 30 feet wide.

This floor may be divided into slabs each 15 feet by 10 feet. (1) By placing main beams across the building at 15 feet centers, and secondary beams at 10 feet centers, as Fig. 80. (2) By placing a main beam down the center supported on two columns, and secondary beams across to the side walls, as Fig. 81.

If the secondary beams in either plan are spaced at $7\frac{1}{2}$ feet centers, making 16 feet floor slabs instead of 12 feet, the reinforcement need run one way only ; this is often desirable as it avoids complication of rods, or it enables a reinforcement to be used similar to that known as triangle mesh, in which the main reinforcement runs one way only.

The arrangement as Fig. 80 will necessitate very heavy main beams, which cannot be avoided if columns are not to be used. For comparison, the beams are here designed for both plans.

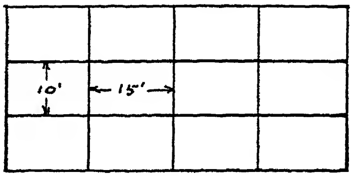


FIG. 80.

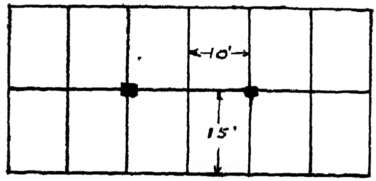


FIG. 81.

Taking the plan as Fig. 80, First design the slabs in the same manner as those of Example XIX. The length being less than twice the breadth, they will require reinforcement on both directions.

Providing for stone concrete 6 inches thick, and a safe live load of 250 lb. per square foot, the total load per square foot will be $250 + \frac{6}{12} \times 150 = 325$ lb.

The bending moment, taking 1 foot of the length first, will be $\frac{W L}{12}$ multiplied by the coefficient f, see page 127,

$$f = \frac{0{\cdot}05\ B + 0{\cdot}45\ L}{B} = \frac{0{\cdot}05 \times 10 + 0{\cdot}45 \times 15}{10} = 0{\cdot}725\ ;$$

hence, $$M = \frac{325 \times 10 \times 10 \times 12 \times 0{\cdot}725}{12} = 23\,562 \text{ inch-lb.}$$

For the economic section,

$$d = \sqrt{\frac{M}{95\ b}} = \sqrt{\frac{23\ 562}{95 \times 12}} = 4\tfrac{1}{2} \text{ inches.}$$

Allowing 1 inch cover, the full thickness of slab will be $5\frac{1}{2}$ inches.

$a = p\ b\ d = 0{\cdot}006\ 75 \times 12 \times 4{\cdot}5 = 0{\cdot}364$ square inch, say two bars $\frac{7}{16}$-inch square $= 0{\cdot}38$ inch.

Taking 1 foot of the width for the longitudinal reinforcement,

$$M = \frac{W\ L\ r}{12},\ \text{and}\ r = \frac{0{\cdot}95\ B - 0{\cdot}45\ L}{B},\ \text{(see page 127)},$$

$$= \frac{0{\cdot}95 \times 10 - 0{\cdot}45 \times 10}{10} = 0{\cdot}275.$$

$$\text{Hence, } M = \frac{325 \times 15 \times 15 \times 12 \times 0{\cdot}275}{12} = 20\ 109\ \text{inch-lb.}$$

For the same stresses the depth of the rods must be :—

$$\sqrt{\frac{M}{95\ b}} = \sqrt{\frac{20\ 109}{95 \times 12}} = 4{\cdot}2\ \text{inches},$$

and a must be $0{\cdot}006\ 75 \times 4{\cdot}2 \times 12 = 0{\cdot}34$ square inch. But to avoid error in placing, the same size rods as across the slab would be used ; they can, however, be placed further apart, the correct spacing being,

$\dfrac{a}{p\ d}$ (see page 95) $= \dfrac{0{\cdot}1914}{0{\cdot}006\ 75 \times 4{\cdot}2} = 6{\cdot}75$ ~~square~~ inches.

Of course, for practical reasons they will be placed directly on top of the bottom rods, which is about one inch lower than is really necessary ; this, however, is an advantage for it will reduce c, this way of the slabs, to about 570 lb.

To effect the strictest economy for the longitudinal reinforcement, the amount required can be determined by the formula, $a = \dfrac{M}{0{\cdot}88\ t\ d}$, where d, in this case equals $4\frac{1}{2}$ inches less half the diameter of the rods, it will therefore be $4\frac{1}{4}$ inches.

$$\text{Then}\quad a = \frac{20\ 109}{0{\cdot}88 \times 16\ 000 \times 4{\cdot}25} = 0{\cdot}336\ \text{square inch.}$$

For the beams at $7\frac{1}{2}$ feet centers,

$$M = \frac{W\ L}{12} = \frac{325 \times 7{\cdot}5 \times 7{\cdot}5 \times 12}{12} = 18\,281 \text{ inch-lb.}$$

$$d = \sqrt{\frac{18\ 281}{95 \times 12}} = 4 \text{ inches.}$$

$a = 0{\cdot}006\ 75 \times 12 \times 4 = 0{\cdot}324$ square inch, to run across the slab only, with distribution rods about 18 inches apart.

Now consider the tee beams. The load for each will be one-half the load for the bay on one side multiplied by the coefficient f, plus the weight of the leg of the beam. The remainder of the load of the floor slab is transmitted to the end supports, and will be taken as a distributed load on the main beams. Assuming the leg will be about 6 inches by 9 inches, its weight will be $150 \times 15 \times 0{\cdot}5 \times 0{\cdot}75 = 844$ lb. The total load will then be $844 + 325 \times 10 \times 15 \times 0{\cdot}725 = 36\ 188$ lb.

As these are continuous beams supported on main beams $M = \frac{W\ L}{12}$ at the center, and $\frac{W\ L}{24}$ at the ends

Then $$M = \frac{36\ 188 \times 15 \times 12}{12} = 542\ 820 \text{ inch-lb.,}$$

at the center.

The slabs being reinforced parallel to the beams, the width of the table must not exceed one-fourth of the slab on each side, allowing for this amount, B will equal 60 inches. By the formula as for rectangular beams and slabs, but substituting B for b, we have :—

$$d = \sqrt{\frac{M}{95\ B}} = \sqrt{\frac{542\,820}{95 \times 60}} = 9{\cdot}76 \text{ inches.}$$

$a = p\,b\,d$. Again substituting B for b, we get $a = 0{\cdot}006\ 75 \times 60 \times 9{\cdot}76 = 3{\cdot}95$ square inches, say 4 bars 1 inch

square. Allowing $1\frac{1}{2}$ inches from the axis of the steel to the bottom of the beam, the full depth will be $11\frac{1}{4}$ inches, or 6 inches projection below the slab.

As previously explained, for stresses of 600 and 16 000 $n = 0{\cdot}36\ d = 0.36 \times 9{\cdot}75 = 3{\cdot}5$ inches, which is less than the effective depth of the slab ; consequently, the beam will do as designed.

For the breadth of the leg we need only consider what width is necessary to properly space the bars, and to comply with the conditions given on page 141, for this portion of concrete is not considered in the strength, the whole being below the axis. To provide room for 4 bars the breadth must be at least 10 inches, which is also sufficient to satisfy the other condition.

If the neutral axis falls below the slab the above formulas will not be correct for designing the depth and reinforcement ; this, however, is only likely to occur when the floor is designed for an exceptionally heavy load and with a thin slab, or when the table of the beam is taken much less than in this example. If such a case occurs, it is best to assume the depth or reinforcement, or both, then check the stresses by the formulas on page 121, and if necessary adjust the dimensions accordingly.

The beams in the present case being continuous, their strength at the supports must be investigated, for here the stresses are reversed, the compression being at the bottom and the tension at the top ; we thus have an inverted tee beam and therefore lose the advantage of the table in resisting compression ; we have then, in effect, a rectangular beam, and being supported on beams the bending moment will be $\frac{W\ L}{24}$, or one-half that for the center = 271 410 inch-lbs.

By bending up two of the bars to within $1\frac{1}{2}$ inches of the top, and running the other two straight through, we get a doubly reinforced beam, effective depth 10 inches, breadth 10 inches, with an area of steel in compression, and in tension, equal to 2 square inches.

Checking the stresses by the formulas for doubly reinforced beams.

$$n = \sqrt{\frac{2\ m\ (a\ d + A_c\ y)}{b} + \left[\frac{m\ (A_c + a)}{b}\right]^2} - \frac{m\ (A_c + a)}{b}$$

$$\sqrt{\frac{30\ (2 \times 10 + 2 \times 1 \cdot 5)}{10} + \left(\frac{15 \times 4}{10}\right)^2} - \frac{15 \times 4}{10}$$

$$= \sqrt{105} - 6 = 4 \cdot 246 \text{ inches.}$$

$$c = \frac{2\ M}{b\ n\ \left(d - \frac{n}{3}\right) + 2\ A_c\, m\ \frac{n - y}{n}\,(d - y)} =$$

$$\frac{2 \times 271\ 410}{10 \times 4 \cdot 246\left(10 - \frac{4 \cdot 246}{3}\right) + 2 \times 2 \times 15\left(\frac{4 \cdot 246 - 1 \cdot 5}{4 \cdot 246}\right)(10 - 1 \cdot 5)}$$

$$= \frac{542\ 820}{694} = 782 \text{ lb.}$$

$$t = \frac{c\ m\ (d - n)}{n} = \frac{782 \times 15\ (10 - 4 \cdot 24)}{4 \cdot 24} = 15\ 935 \text{ lb.}$$

This shows the concrete to be considerably over-stressed but the bars cannot be better arranged, therefore the depth must be increased towards the ends, or an additional bar must be placed in the bottom to take up the excess compression.

The former method can only be adopted where the appearance is not an objection.

By placing another inch bar in the bottom we get :—

Area of steel in compression 3 square inches.

Area of steel in tension 2 square inches.

$$n = \sqrt{\frac{30\ (2 \times 10 + 3 \times 1\tfrac{1}{2})}{10} + \left(\frac{15 \times 5}{10}\right)^2} - \frac{15 \times 5}{10} = 3 \cdot 89.$$

$$c = \frac{542\ 820}{10 \times 3{\cdot}89\left(10 - \frac{3{\cdot}89}{3}\right) + 2 \times 3 \times 15\left(\frac{3{\cdot}89 - 1{\cdot}5}{3{\cdot}89}\right) 8{\cdot}5}$$

$$= 671 \text{ lb.}$$

$$t = \frac{671 \times 15 \times 6{\cdot}11}{3{\cdot}89} = 15\ 809 \text{ lb.}$$

The concrete is still a trifle overstressed. A further adjustment can be made if considered necessary.

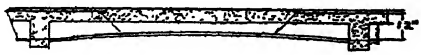


FIG. 82.

Without an extra bar, but by increasing the effective depth to 12 inches, and bending down the two bars, as Fig. 82, we get the following :—

$$n = \sqrt{\frac{30\,(2 \times 12 + 2 \times 1{\cdot}5)}{10} + \left(\frac{15 \times 4}{10}\right)^2} - \frac{15 \times 4}{10}$$

$$= \sqrt{117} - 6 = 4{\cdot}82 \text{ inches.}$$

$$c = \frac{542\ 820}{10 \times 4{\cdot}82\left(12 - \frac{4{\cdot}82}{3}\right) + 2 \times 2 \times 15\left(\frac{4{\cdot}82 - 1{\cdot}5}{4{\cdot}82}\right)(12 - 1{\cdot}5}$$

$$= \frac{542\ 820}{935} = 580 \text{ lb.}$$

$$t = \frac{580 \times 15\ (12 - 4{\cdot}82)}{4{\cdot}82} = 12\ 960 \text{ lb.}$$

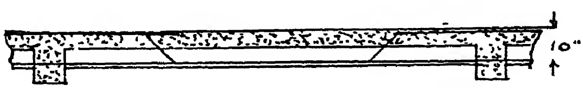


FIG. 83.

We may therefore design the tee beams as Fig. 82 or Fig. 83.

MAIN BEAMS.

These are 30 feet long, with a concentrated load 10 feet from each end equal to the reactions of the tee beams on each side ; the load at each point thus equals the load for one beam = 36 188 lb. To this must be added the uniformly distributed load of the beam itself, and the remainder of the load for the slabs, not taken with the tee beams, that is the portion directly transmitted to the main beams by the ends of the slabs resting upon them. Allowing for the beams to be about 4 feet by 2 feet 3 inches, its weight will be, $4 \times 2{\cdot}25 \times 30 \times 150 = 40\ 500$ lbs.

The load transmitted to the beam directly by the slab is equal to the load for three slabs multiplied by the coefficient r, $= 3 \times 325 \times 10 \times 15 \times 0{\cdot}275 = 40\ 218$ lb.

The distributed load therefore equals $40\ 500 + 40\ 218 = 80\ 718$ lb.

The bending moment at the center from the concentrated loads will be the reaction of one end multiplied by the distance to the load (*vide* Fig. 64) $= 36\ 188 \times 10 \times 12 = 4\ 342\ 560$ lb.

The bending moment of the distributed load =

$$\frac{WL}{8} = \frac{80\ 718 \times 30 \times 12}{8} = 3\ 632\ 310 \text{ inch-lb.}$$

Hence $M = 4\ 342\ 560 + 3\ 632\ 310 = 7\ 974\ 870$ inch-lb.

For a singly reinforced beam

$$d = \sqrt{\frac{M}{95\,b}}.$$

Taking b as $0{\cdot}6\,d$, then

$$d = \sqrt[3]{\frac{M}{0{\cdot}6 \times 95}} = \sqrt[3]{\frac{7\ 974\ 870}{0{\cdot}6 \times 95}} = 52 \text{ inches.}$$

$b = 0{\cdot}6\,d = 0{\cdot}6 \times 52 = 31{\cdot}2$ inches. $a = p\,b\,d = 0{\cdot}006\,75 \times 31{\cdot}2 \times 52 = 10{\cdot}95$ square inches. Say 7 bars $1\frac{1}{4}$ inch square.

By this we see that to use beams with single reinforcement, proportioned to develop the maximum allowable stresses they would require to have an effective depth of 52 inches, with a breadth of $31\frac{1}{4}$ inches, which would be large heavy beams.

The size can be reduced by using compression bars, thus making a doubly reinforced beam, as follows:— Limiting the effective depth to 3 feet, the breadth to 20 inches, and allowing 2 inches cover to the steel, the full section will be 38×20 inches. Its weight will be

$$\frac{38 \times 20}{144} \times 30 \times 150 = 23\,750 \text{ lb.}$$

The bending moment for the distributed load will then equal

$$\frac{(23\,750 + 40\,218)\;30 \times 12}{8} = 2\,878\,560 \text{ inch-lb.}$$

Then total M equals $4\,342\,560 + 2\,878\,560 = 7\,221\,120$ inch-lb.

Now design the reinforcement as in Example IX. $a = p\ b\ d = 0{\cdot}006\,75 \times 20 \times 36 = 4{\cdot}86$ square inches. The bending moment this will resist equals

$$t\,a\left(d - \frac{n}{3}\right),$$

$$\text{and } n = \frac{m\,c\,d}{t + m\,c} = \frac{15 \times 600 \times 36}{16\,000 + 15 \times 600} = 12{\cdot}96 \text{ inches.}$$

Hence

$$\text{M} = 16\,000 \times 4{\cdot}86\left(36 - \frac{12{\cdot}96}{3}\right) = 2\,463\,437 \text{ inch-lb.}$$

This leaves a balance for additional reinforcement of 7 221 120 — 2 463 437 = 4 757 683 inch-lb.

The moment of tension will be $a' t (d - y)$. Equating this with the excess bending moment,

$$a' = \frac{M}{t(d-y)}.$$

Placing the compression bars 2 inches from the top,

then $$a' = \frac{4\ 757\ 683}{16\ 000\ (36-2)} = 8{\cdot}75 \text{ square inches},$$

and

$$A_c = a' \frac{d-n}{n-y} = \frac{8{\cdot}75\ (36-12{\cdot}96)}{12{\cdot}96-2} = 18{\cdot}39 \text{ square inches.}$$

We therefore require 18·39 square inches of steel for the compression bars, and 4·86 + 8·75, say 13·61 square inches for the tension bars. Say 7 bars 1 $\frac{7}{16}$-inch square in the bottom, and 7 bars 1$\frac{5}{8}$ inch square in the top.

For the floor as plan, Fig. 81, the slab will do as designed for Fig. 80, also the secondary beams, but for the beams that intersect over the columns the bending moment should be taken as previously explained.

The main beams can be taken as fixed at the ends with a central load equal to the reactions of the two secondary beams, which will be half the load on each, a total of 36 188 lb. To this must be added the uniformly distributed load of the beam itself, and the remainder of the slab load not taken with the tee beams. Assuming the beam will be about 2 feet 6 inches by 1 foot 6 inches, its weight will be $2{\cdot}5 \times 1{\cdot}5 \times 20 \times 150$ lb. = 11 250 lb. The load transmitted to the beam directly by the slab is equal to the load for two slabs multiplied by the coefficient r, $= 2 \times 325 \times 10 \times 15 \times 0{\cdot}275 = 26\ 812$ lb. The bending moment for the concentrated load, for the

center and ends, may be taken as $\frac{W L}{8}$ and that for the distributed loads will be $\frac{W L}{12}$, then the total bending moment will be

$$\frac{36\ 188 \times 20 \times 12}{8} + \frac{(11\ 250 + 26\ 812)\ 20 \times 12}{12}$$

$$= 1\ 085\ 640 + 761\ 250 = 1\ 846\ 890 \text{ inch-lb.}$$

For a singly reinforced beam,

$$d = \sqrt{\frac{M}{95\ b}}.$$

Taking b as $0{\cdot}6\ d$, then

$$d = \sqrt[3]{\frac{1\ 846\ 890}{0{\cdot}6 \times 95}} = \sqrt[3]{32\ 400}, \text{ say } 31{\cdot}88 \text{ inches.}$$

$b = 0{\cdot}6\ d$, say $19{\cdot}13$ inches, and $a = p\ b\ d = 0{\cdot}006\ 75 \times 19{\cdot}13 \times 31{\cdot}88 = 4{\cdot}12$ square inches. Say 4 bars 1 inch square. A beam with single reinforcement will therefore require an effective depth of 2 feet 8 inches, breadth 1 foot 7 inches, and 4·12 square inches of steel.

FOR A DOUBLY REINFORCED BEAM.

Taking d as 24 inches, b as 14 inches, and designing as in the previous case. Weight of beam equals

$$\frac{26}{12} \times \frac{14}{12} \times 20 \times 150 = 7\ 583 \text{ lb.}$$

$$M = 1\ 085\ 640 + \frac{(7\ 583 + 26\ 812)\ 20 \times 12}{12} = 1\ 773\ 540 \text{ inch-}$$

lb. $a = 0{\cdot}006\ 75 \times 24 \times 14 = 2{\cdot}268$ square inches. The

bending moment this will resist

$$= t\,a\left(d - \frac{n}{3}\right),$$

and $n = 0{\cdot}36\,d = 8{\cdot}64$ inches.

Therefore,

$$M = 16\,000 \times 2{\cdot}268 \left(24 - \frac{8{\cdot}64}{3}\right) = 766\,403 \text{ inch-lb.}$$

This leaves a balance for additional reinforcement of 1 773 540 — 766 403 = 1 007 137 inch-lb.

$$a' = \frac{M}{t\,(d - y)}.$$

Taking y as 2 inches, then

$$a' = \frac{1\,007\,137}{16\,000 \times 22} = 2{\cdot}86 \text{ square inches.}$$

$$A_c = a'\,\frac{d - n}{n - y} = \frac{2{\cdot}86\,(24 - 8{\cdot}64)}{8{\cdot}64 - 2} = 6{\cdot}62 \text{ square inches.}$$

Total area of steel in tension = 5·13 square inches.
Total area of steel in compression = 6·62 square inches.
Say 5 tension bars and 6 compression bars, each $1\frac{1}{16}$ inch square.

Designing this beam by the formulas given on page 120 we get :—

$$d = \sqrt[3]{\frac{1\,773\,540}{152 \times 0{\cdot}6}} = 26{\cdot}9 \text{ inches.} \quad b = 0{\cdot}6\,d = 0{\cdot}6 \times 26{\cdot}9$$

$$= 16{\cdot}14 \text{ inches.} \quad a = p\,b\,d = 0.0108 \times 16{\cdot}14 \times 26{\cdot}9 =$$

4·69 square inches, and $A_c = a = 4{\cdot}69$ square inches, which must not be placed further from the compression surface than $\frac{1}{3}\ n$.

These beams, with the tee-beams of Fig. 82, will give the best case. The longitudinal and transverse sections of the whole floor will be as Figs. 84 and 85.

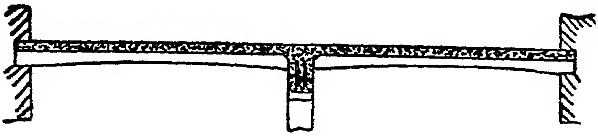


Fig. 85.

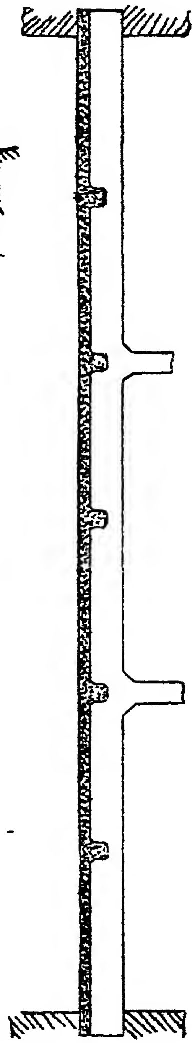


Fig. 84.

FLAT SLAB FLOORS.

This system of floor construction is the most difficult one for the inexperienced to design, but its advantages over the beam system for industrial and similar buildings is giving it rapidly increasing popularity. It is claimed to be the most economical floor in form construction, and in the installation of the sprinkler system of fire protection, by virtue of the flat ceiling, see Fig. 86 ; also in that there is a saving in storey height, due to absence of beams, amounting to approximately one whole storey in 8 or 9, depending upon the ceiling heights adopted, this reduction in the total height of a building effects a very considerable saving in the main walls, stairways, elevator structure, heating, plumbing, and electrical installations. Other claimed advantages are :—(1) Better light and ventilation, there being no beams to cast shadows, interfere with the diffusion of light or to prevent free circulation of air currents. (2) More

fire resistance ; as there are no projecting corners to be exposed to the action heat and water, as in beam floors. (3) A considerable saving in time of construction, the storeys being less in height, the floor more simple in construction, and more speedier in form work, than for any other kind of floor.

TYPES OF FLAT SLAB CONSTRUCTION.

In this system there is no standard method of distribution, or placing of the bars, nor is there any standard

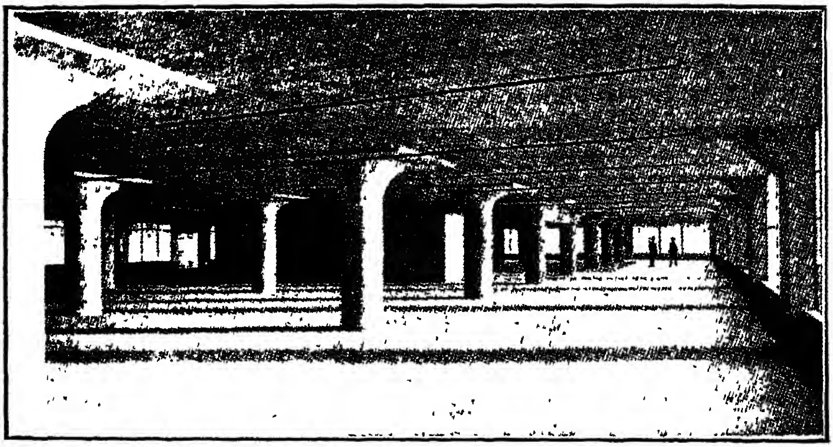


FIG. 86.

Flat Slab Floor, showing the clear flat ceiling and the arrangement of column caps, drop panels, and the sprinkler system of fire protection.

formula for designing, various formulas and methods are adopted according to the ideas of the designer, developed through laboratory work and analysis determined by tests made on completed floors, but all formulas are more or less empirical as it is a type of structure in which the stresses are impossible to accurately determine ; no adequate theoretical analysis of the stresses has ever been given ; consequently, there is much uncertainty as to the distribution of the stresses within the slab.

The slab may be of uniform thickness throughout, or it may be thickened a few feet out, and around the

column head, forming what is termed a drop panel; or it may be thickened between the columns to appear like a wide shallow beam which forms a central sunk panel. The method of reinforcing may be classed under four headings. (1) The two-way system, in which the reinforcement is placed in two directions, at right angles

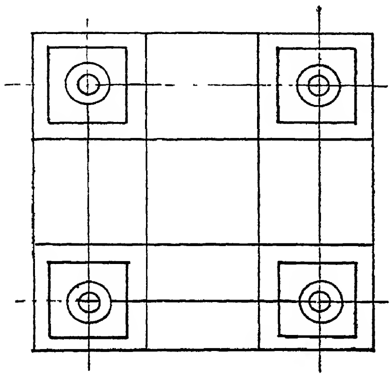


FIG. 87.

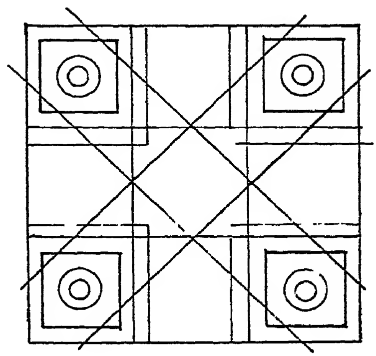


FIG. 88.

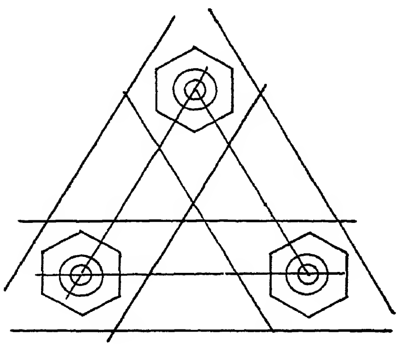


FIG. 89.

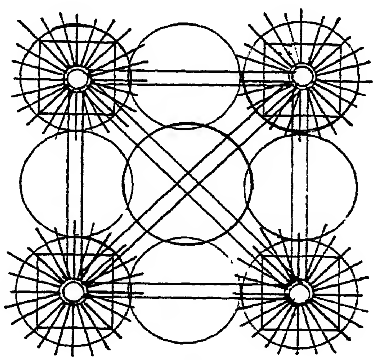


FIG. 90.

to one another and parallel to the sides of the panel formed by lines joining the columns. (2) The four-way system, in which the reinforcement is placed over a certain width of the slab between the columns, in the same directions as in the two-way system, and in addition two widths of reinforcement placed diagonally between the columns. (3) The three-way system, in

which the columns are arranged at the apexes of equalateral triangles, and the reinforcement placed in three directions passing directly over the columns, which have flaring caps with hexagonal or circular drops. (4) The circumferential system, in which circular bands of reinforcement are placed around the columns with bars radiating from the column centers. Circular bands of reinforcement are also placed at the center of the sides joining the column head ; these bands overlap those around the column heads. The centre of the panel is reinforced in a similar manner.

These four systems are illustrated in Figs. 87, 88, 89, 90.

There are other methods of arranging the reinforcement which are either a modification of one, or a combination of two, of those described.

Several methods are patented, of which the most generally known is the C. A. P. Turner Mushroom System. The latest improvement of this is known as the Spiral Mushroom System, illustrated in Fig. 91, in which the various stages of placing the steel are shown. Stage one shows the frame of spider on which the flat spirals are placed, the spider keeping the spirals at the proper height near the top of the slab. In the second stage, series of rods of a width approximately two-thirds the diameter of the spiral are placed in two directions parallel to the column lines, these rods being at the top of the slab over the spirals and dipping sharply downward to the bottom of the slab beyond the line of inflection. Then diagonal strip from mid-span to mid-span of the sides are placed in the bottom throughout, as shown in the third stage. And finally in the fourth stage it is shown how parallel rods in two directions fill in the central area, each set of rods rising to the top of the slab over the saddle back areas and dropping to the bottom therefrom, and continuing through the bottom of the slab in the central half of the panel.

It thus appears that the entire area of the slab is completely covered by parallel rods in two directions, which arrangement is effected in four stages as just

described in order to be able to place these rods at proper levels with respect to the rest of the reinforcement in a convenient manner.

The chief advantage of the Spiral Mushroom over the original form of construction, lies in the reduction of the thickness of the mat of slab steel over the supports. This

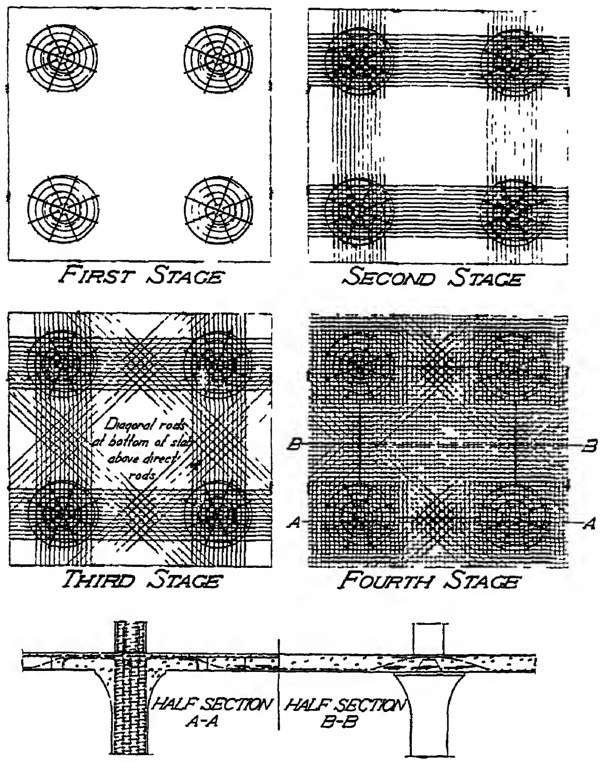


FIG. 91.
Showing the various stages of placing the steel in the C. A. P. Turner Spiral Mushroom System of flat slab floor.

thickness is reduced in two layers of slab rods and a spiral rod which is generally about the size of the slab rods. There are accordingly only three thicknesses of steel in place of five layers in the original system.

The limiting minimum thickness of the slab for plain rods is taken as $\frac{L}{48}$ plus an amount necessary for massing

of the steel, embedment, and fireproofing, which amount is taken as 2 inches for $\frac{3}{8}$ inch or smaller rods, $2\frac{1}{4}$ inches for $\frac{7}{16}$ inch rods, $2\frac{1}{2}$ inches for $\frac{1}{2}$ inch rods, and 3 inches for $\frac{7}{8}$ inch rods.

L equals the span between column centers. For rectangular panels L is for the longer span.

For panels supported on beams or walls, the embedment thickness is taken 1 inch less than given above, but if the panels are rectangular, L is taken as $\frac{(a^2+b^2)}{a+b}$ where a and b are the spans of the two directions respectively.

The two-way system appears to be the most generally used, probably on account of its comparative simplicity of design and construction. The four-way system is the next most used.

Various authorities, and city building ordinances, differ in their requirements for this class of floor construction, any one of the following, however, may be considered to meet the conditions of any case, in the most logical and satisfactory manner. Many floors have been designed by each of them, and they all appear to have given results consistent with safety and economical construction.

AMERICAN JOINT COMMITTEE ON CONCRETE AND REINFORCED CONCRETE.

The co-efficients and moments given relate to uniformly distributed loads.

(a) **Column Capital.** The moment of the external forces which the slab is called upon to resist is dependent upon the size of the capital; the section of the slab immediately above the upper periphery of the capital carries the highest amount of punching shear; and the bending moment developed in the column by an eccentric or unbalanced loading of the slab is greatest at the under surface of the slab. Generally the horizontal section of the column capital should be round, or square

with rounded corners. In oblong panels the section may be oval, or oblong, with dimensions proportional to the panel dimensions. For computation purposes, the diameter of the column capital will be considered to be measured where its vertical thickness is at least 1½ inches provided the slope of the capital below this point nowhere makes an angle with the vertical of more than 45 degrees. In case a cap is placed above the column capital, the part of this cap within a cone made by extending the lines of the column capital upward at an angle of 45 degrees to the bottom of the slab or dropped panel may be considered as part of the column capital in determining the diameter for design purposes. Without attempting to limit the size of the column capital for special cases, it is recommended that the diameter of the column capital (or its dimension parallel to the edge of the panel) generally be made not less than one-fifth of the dimension of the panel from center to center of adjacent columns. A diameter equal to 0·225 of the panel length has been used quite widely and acceptably. For heavy loads or large panels special attention should be given to designing and reinforcing the column capital with respect to compressive stresses and bending moments. In case of heavy loads or large panels, and where the conditions of the panel loading or variations in panel length or other conditions cause high bending stresses in the column, and also for column capitals smaller than the size herein recommended, especial attention should be given to designing and reinforcing the column capital with respect to compression and to rigidity of connection to floor slab.

(b) **Dropped Panel.** In one type of construction the slab is thickened throughout an area surrounding the column capital. The square or oblong of thickened slab thus formed is called a dropped panel or drop. The thickness and the width of the dropped panel may be governed by the amount of resisting moment to be provided (the compressive stress in the concrete being dependent upon both thickness and width), or its thickness may be governed by the resistance to shear required at the edge of the column capital and its width by the

allowable compressive stresses and shearing stresses in the thinner portion of the slab adjacent to the dropped panel. Generally, however, it is recommended that the width of the dropped panel be at least four-tenths of the corresponding side of the panel as measured from center to center of columns, and that the offset in thickness be not more than five-tenths of the thickness of the slab outside the dropped panel.

(c) **Slab Thickness.** In the design of a slab, the resistance to bending and to shearing forces will largely govern the thickness, and, in the case of large panels with light loads, resistance to deflection may be a controlling factor. The following formulas for minimum thicknesses are recommended as general rules of design when the diameter of the column capital is not less than one-fifth of the dimension of the panel from center to center of adjacent columns, the large dimension being used in the case of oblong panels. In the following formulas $t =$ total thickness of slab in inches. $L =$ panel length in feet. $w =$ sum of live load and dead load in pounds per square foot.

For a slab without dropped panels, minimum $t = 0{\cdot}024 L\sqrt{w} + 1\frac{1}{2}$; for a slab with dropped panels, minimum $t = 0{\cdot}2\, L\sqrt{w} + 1$; for a dropped panel of which the width is four-tenths of the panel length, minimum $t = 0{\cdot}03\, L\sqrt{w} + 1\frac{1}{2}$.

In no case should the slab thickness be made less than 6 inches, nor should the thickness of a floor slab be made less than $\frac{1}{32}$ of the panel length, nor the thickness of a roof slab less than $\frac{1}{40}$ of the panel length.

(d) **Bending and Resisting Moments in Slabs.** If a vertical section of a slab be taken across a panel along a line midway between columns, and if another section be taken along an edge of the panel parallel to the first section, but skirting the part of the periphery of the column capitals at the two corners of the panels, the moment of the couple formed by the external load on the half panel, exclusive of that over the column capital (sum of live and dead load), and the resultant of the external shear or reaction at the support at the two

column capitals (see Fig. 92), may be found by ordinary static analysis. It will be noticed that the edges of the area here considered are along lines of zero shear except around the column capitals. This moment of external forces acting on the half panel will be resisted by the numerical sum of (a) the moment of the internal stresses at the section of the panel midway between columns (positive resisting moment), and (b) the moment of the internal stresses at the section referred to at the end of the panel (negative resisting moment). In the curved

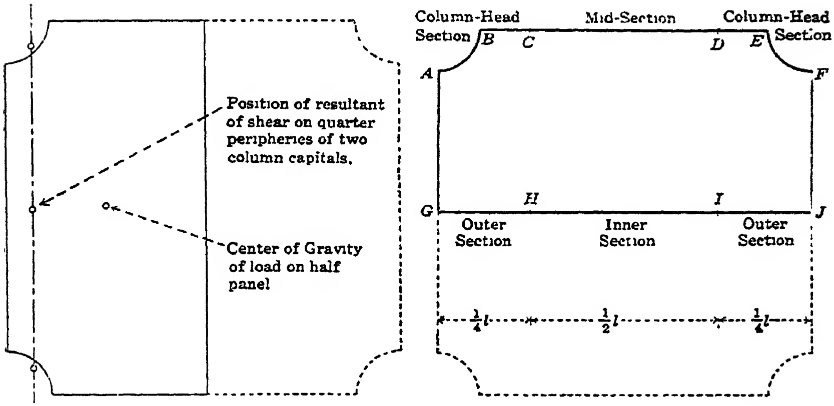


FIG. 92

portion of the end section (that skirting the column), the stresses considered are the components which act parallel to the normal stresses on the straight portion of the section. Analysis shows that, for a uniformly distributed load, and round columns, and square panels, the numerical sum of the positive moment and the negative moment at the two sections named is given quite closely by the equation $M = \frac{1}{8} w\, l \left(l - \frac{2d}{3}\right)^2$. In this formula and in those which follow relating to oblong panels :—

w = sum of the live and dead load per unit of area.

l = side of a square panel measured from center to center of columns.

l_1 = one side of the oblong panel measured from center to center of the columns.

l_2 = other side of oblong panel measured in the same way.

d = diameter of the column capital.

M_x = numerical sum of positive moment and negative moment in one direction.

M_y = numerical sum of positive moment and negative moment in the other direction.

For oblong panels, the equations for the numerical sums of the positive moment and the negative moment at the two sections named become,

$$M_x = \tfrac{1}{8} w\, l_2 \left(l_1 - \frac{2\,d}{3}\right)^2. \quad M_y = \tfrac{1}{8} w\, l_1 \left(l_2 - \frac{2\,d}{3}\right)^2.$$

Where M_x is the numerical sum of the positive moment and the negative moment for the sections parallel to the dimensions l_2, and M_y is the numerical sum of the positive moment and the negative moment for the sections parallel to the dimensions l_1.

What proportion of the total resistance exists as positive moment, and what as negative moment is not readily determined. The amount of the positive moment and that of the negative moment may be expected to vary somewhat with the design of the slab. It seems proper, however, to make the division of total resisting moment in the ratio of $\frac{3}{8}$ for the positive moment to $\frac{5}{8}$ for the negative moment.

With reference to variations of stress along the sections it is evident from conditions of flecture that the resisting moment is not distributed uniformly along either the section of positive moment or that of negative moment. As the law of the distribution is not definitely known, it will be necessary to make an empirical apportionment along the sections; and it will be considered sufficiently accurate generally to divide the sections into two parts and to use an average value over each part of the panel section.

The relatively large breadth of structure in a flat slab makes the effect of local variations in the concrete less than would be the case for narrow members like beams.

The tensile resistance of the concrete is less affected by cracks. Measurements of deformations in buildings under heavy load indicate the presence of considerable tensile resistance in the concrete, and the presence of this tensile resistance acts to decrease the intensity of the compressed stresses. It is believed that the use of moment coefficients somewhat less than those given in a preceding paragraph as derived by analysis is warranted, the calculations of resisting moment and stresses in concrete and reinforcement being made according to the assumptions specified in this report, and no change being made in the values of the working stresses ordinarily used. Accordingly, the values which are recommended for use are somewhat less than those derived by analysis. The values given may be used when the column capitals are round, oval, square or obling.

(e) **Names of Moment Sections.** For convenience, that portion of the section across a panel along a line midway between columns which lies between the middle two quarters of the width of the panel (HI, Fig. 92), will be called the inner section, and that portion in the two outer quarters of the width of the panel (GH and IJ, Fig. 92), will be called the outer sections. Of the section which follows a panel edge from column capital to column capital, and which includes the quarter peripheries of the edges of two column capitals, that portion within the middle two quarters of the panel width (CD, Fig. 92), will be called the mid-section, and the two remaining portions (ABC and DEF, Fig. 92), each having a projected width equal to one-fourth of the panel width, will be called the column-head sections.

(f) **Positive Moment.** For a square interior panel, it is recommended that the positive moment for a section in the middle of a panel extending across its width be taken as $\frac{1}{25}\ w\ l\left(l-\frac{2c}{3}\right)^2$. Of this moment, at least 25 per cent. should be provided for in the inner section; in the two outer sections of the panel at least 55 per cent. of the specified moment should be provided for in slabs

not having dropped panels, and at least 60 per cent. in slabs having dropped panels, except that in calculations to determine necessary thickness of slab away from the dropped panel at least 70 per cent. of the positive moment should be considered as acting in the two outer sections.

(g) **Negative Moment.** For a square interior panel, it is recommended that the negative moment for a section which follows a panel edge from column capital to column capital and which includes the quarter peripheries of the edges of the two column capitals (the section altogether forming the projected width of the panel) be taken as $\frac{1}{15}\,w\,l\left(l-\frac{2c}{3}\right)^2$. Of this negative moment, at least 20 per cent. should be provided for in the mid-section and at least 65 per cent. in the two column-head sections of the panel, except that in slabs having dropped panels at least 80 per cent. of the specified negative moment, should be provided for in the two column-head sections of the panel.

(h) **Moments for Oblong Panels.** When the length of a panel does not exceed the breadth by more than 5 per cent., computation may be made on the basis of a square panel with sides equal to the mean of the length and the breadth.

When the long side of an interior oblong panel exceeds the short side by more than one-twentieth and by not more than one-third of the short side, it is recommended that the positive moment be taken as $\frac{1}{25}\,w\,l_2\left(l_1-\frac{2c}{3}\right)^2$ on a section parallel to the dimension l_2, and $\frac{1}{25}\,w\,l_1\left(l_2-\frac{2c}{3}\right)^2$ on a section parallel to dimension l_1; and that the negative moment be taken as $\frac{1}{15}\,w\,l_2\left(l_1-\frac{2c}{3}\right)^2$ on a section at the edge of the panel corresponding to the dimension l_2, and $\frac{1}{15}\,w\,l_1\left(l_2-\frac{2c}{3}\right)^2$ at a section in the

other direction. The limitations of the apportionment of moment between inner section and outer section and between mid-section and column-head sections may be the same as for square panels.

(i) **Wall Panels.** The co-efficient of negative moment at the first row of columns away from the wall should be increased 20 per cent. over that required for interior panels, and likewise the co-efficient of positive moment at the section half-way to the wall should be increased by 20 per cent. If girders are not provided along the wall or the slab does not project as a cantilever beyond the column line, the reinforcement parallel to the wall for the negative moment in the column-head section and for the positive moment in the outer section should be increased by 20 per cent. If the wall is carried by the slab this concentrated load should be provided for in the design of the slab. The co-efficient of negative moments at the wall to take bending in the direction perpendicular to the wall line may be determined by the conditions of restraint and fixedness as found from the relative stiffness of columns and slab, but in no case should it be taken as less than one-half of that for interior panels.

(j) **Reinforcement.** In the calculation of moments all the reinforcing bars which cross the section under consideration and which fulfill the requirements given under paragraph (l) of this chapter may be used. For a column-head section reinforcing bars parallel to the straight portion of the section do not contribute to the negative resisting moment for the column-head section in question. In the case of four-way reinforcement the sectional area of the diagonal bars multiplied by the sine of the angle between the diagonal of the panel and straight portion of the section under consideration may be taken to act as reinforcement in a rectangular direction.

(k) **Point of Inflection.** For the purpose of making calculations of moments at sections away from the sections of negative and positive moment already specified, the point of inflection on any line parallel to a panel edge may be taken as one-fifth of the clear distance on that line between the two sections of negative moment

at the opposite ends of the panel indicated in paragraph (e) of this chapter. For slabs having dropped panels the co-efficient of one-fourth should be used instead of one-fifth.

(l) **Arrangement of Reinforcement.** The design should include adequate provision for securing the reinforcement in place so as to take not only the maximum moments, but the moments at intermediate sections. All bars in rectangular bands or diagonal bands should extend on each side of a section of maximum moment, either positive or negative, to points at least twenty diameters beyond the point of inflection as defined herein or be hooked or anchored at the point of inflection. In addition to this provision bars in diagonal bands used as reinforcement for negative moment should extend on each side of a line drawn through the column center at right angles to the direction of the band at least a distance equal to thirty-five one-hundredths of the panel length, and bars in diagonal bands used as reinforcement for positive moment should extend on each side of a diagonal through the center of the panel at least a distance equal to thirty-five one-hundredths of the panel length ; and no splice by lapping should be permitted at or near regions of maximum stress except as just described. Continuity of reinforcing bars is considered to have advantages, and it is recommended that not more than one-third of the reinforcing bars in any direction be made of a length less than the distance center to center of columns in that direction. Continuous bars should not all be bent up at the same point of their length, but the zone in which this bending occurs should extend on each side of the assumed point of inflection, and should cover a width of at least one-fifteenth of the panel length. Mere draping of the bars should not be permitted. In a four-way reinforcement the position of the bars in both diagonal and rectangular directions may be considered in determining whether the width of zone of bending is sufficient.

(m) **Reinforcement at Construction Joints.** It is recommended that at construction joints reinforcing bars

equal in section to 20 per cent. of the amount necessary to meet the requirements for moments at the section where the joint is made, be added to the reinforcement, these bars to extend not less than 50 diameters beyond the joint on each side.

(n) **Tensile and Compressive Stresses.** The usual method of calculating the tensile and compressive stresses in the concrete and in the reinforcement, based on the assumptions for internal stresses given in this report, should be followed. In the case of the dropped panel, the section of the slab and dropped panel may be considered to act integrally for a width equal to the width of the column-head section.

(o) **Provision for Diagonal Tension and Shear.** In calculations for the shearing stress which is to be used as the means of measuring the resistance to diagonal tension stress, it is recommended that the total vertical shear on two column-head sections constituting a width equal to one-half the lateral dimensions of the panel, for use in the formula for determining critical shearing stresses, be considered to be one-fourth of the total live and dead load on a panel for a slab of uniform thickness, and to be three-tenths of the sum of the live and dead loads on a panel for a slab with dropped panels. The formula for shearing unit stress may then be written $v = \frac{0{\cdot}25\ W}{b\ j\ d}$ for slabs of uniform thickness, and $v = \frac{0{\cdot}30\ W}{b\ j\ d}$ for slabs with dropped panels, where W is the sum of the dead and live load on the panel, b is half the lateral dimension of the panel measured from center to center of the columns, and $j\ d$ is the lever arm of the resisting couple at the section.

The calculation of what is commonly called punching shear may be made on the assumption of a uniform distribution over the section of the slab around the preciphery of the column capital and also of a uniform distribution over the section of the slab around the periphery of the dropped panel, using in each case an amount

of vertical shear greater by 25 per cent. than the total vertical shear on the section under consideration.

(p) **Wall and Openings.** Girders or beams should be constructed to carry walls and other concentrated loads which are in excess of the working capacity of the slab. Beams should also be provided in case openings in the floor reduce the working strength of the slab below the required carrying capacity.

(q) **Unusual Panels.** The co-efficients, apportionments, and thicknesses recommended are for slabs which have several rows of panels in each direction, and in which the size of the panels is approximately the same. For structures having a width of one, two or three panels, and also for slabs having panels of markedly different sizes, an analysis should be made of the moments developed in both slab and columns and the values given herein modified accordingly. Slabs with panelled ceiling or with depressed panelling in the floor are to be considered as coming under the recommendations herein given.

(r) **Bending Moments in Columns.** Provision should be made in both wall columns and interior columns for the bending moment which will be developed by unequally loaded panels, eccentric loading, or uneven spacing of columns. The amount of moment to be taken by a column will depend upon the relative stiffness of the columns and slab, and computations may be made by rational methods, such as the principle of least work, or of slope and deflection. Generally, the larger part of the unequalized negative moment will be transmitted to the columns, and the column should be designed to resist this moment. Especial attention should be given to wall columns and corner columns.

Requirements of American Joint Committee and Building Departments of various American Cities for Reinforced Concrete Flat-slab Floor Construction.

	Joint Committee Washington and Boston	Amer Concrete Institute	Chicago and Los Angeles	Philadelphia and Baltimore	Detroit	St. Louis	Pittsburg	Cincinnati	Indianapolis
Min. size of Column			$\frac{L}{12}$			$\frac{L}{15}$		$\frac{L}{12}$	
Slab thickness	$\frac{L}{32}$ or $\frac{\sqrt{W}}{50}+1''$	$\frac{L}{32}$ or $\frac{\sqrt{W}}{50}+1''$	$\frac{L}{32}$ or $\frac{\sqrt{W}}{44}$ min 6"		$\frac{L}{32}$	$\frac{L}{32}$ or $\frac{\sqrt{W}}{44}$ min. 6"		6" or $\frac{L}{32}$	$\frac{L}{30}$
Column Capital	$\frac{L}{5}$ or 0.225 L	$\frac{L}{5}$	$\frac{L}{4.44}$	$\frac{L}{5}$	$\frac{L}{4.44}$		$\frac{L}{4.44}$	$\frac{L}{4.44}$	$\frac{L}{4.44}$
Drop Panel	$\frac{L}{2.5}$	$\frac{L}{3.33}$	$\frac{L}{3}$	$\frac{L}{2.63}$	$\frac{L}{2.67}$			$\frac{L}{3.33}$	
2-way System Strip A + M	$\frac{WL}{55.5}$	$\frac{WL}{82.2}$	$\frac{WL}{60}$	$\frac{WL}{77.5}$	$\frac{WL}{62}$	$\frac{WL}{66.7}$	$\frac{WL}{64}$	$\frac{WL}{60}$	$\frac{WL}{120}$
Strip A − M	$\frac{WL}{25}$	$\frac{WL}{29.6}$	$\frac{WL}{30}$	$\frac{WL}{31}$	$\frac{WL}{30.5}$	$\frac{WL}{33.3}$	$\frac{WL}{29.5}$	$\frac{WL}{30}$	$\frac{WL}{30}$
Strip B + M	$\frac{WL}{130}$	$\frac{WL}{123}$	$\frac{WL}{120}$	$\frac{WL}{124}$	$\frac{WL}{116}$	$\frac{WL}{133.3}$	$\frac{WL}{64}$	$\frac{WL}{120}$	$\frac{WL}{120}$
Strip B − M	$\frac{WL}{100}$	$\frac{WL}{148}$	$\frac{WL}{120}$	$\frac{WL}{124}$	$\frac{WL}{116}$	$\frac{WL}{133.3}$		$\frac{WL}{120}$	$\frac{WL}{120}$
4-way System Strip A + M	$\frac{WL}{55.5}$	$\frac{WL}{82.2}$	$\frac{WL}{80}$	$\frac{WL}{94}$	$\frac{WL}{100}$	$\frac{WL}{100}$	$\frac{WL}{64}$	$\frac{WL}{80}$	$\frac{WL}{120}$
Strip A − M	$\frac{WL}{25}$	$\frac{WL}{29.6}$	$\frac{WL}{30}$	$\frac{WL}{28.4}$	$\frac{WL}{30}$	$\frac{WL}{50.1}$	$\frac{WL}{29.5}$	$\frac{WL}{30}$	$\frac{WL}{30}$
Strip B + M	$\frac{WL}{130}$	$\frac{WL}{123}$	$\frac{WL}{120}$	$\frac{WL}{144}$	$\frac{WL}{100}$	$\frac{WL}{100}$	$\frac{WL}{64}$	$\frac{WL}{120}$	$\frac{WL}{120}$
Strip B − M	$\frac{WL}{100}$	$\frac{WL}{148}$	$\frac{WL}{120}$	$\frac{WL}{155}$		$\frac{WL}{100}$		$\frac{WL}{120}$	
Wall Column, M			$\frac{WL}{60}$	$\frac{WL}{77.5}$				$\frac{WL}{75}$	

W = Total live and dead load on the panel
L = Length of panel measured centre to centre of columns.
Strips A and B are the same width in all cases, and equals half the span of the panel, center to center of the column See Fig 94.

The following examples are given to elucidate the formulas for designing a flat-slab floor system.

Example XXVII.—Determine the design for a four-way flat slab floor to carry a load of 250 lb. per square foot, including its own weight. The columns to be spaced at 20 feet centers. The design to comply with the requirements of the Chicago and Los Angeles Building Codes.

Diameter of column $= \frac{L}{12} = 20$ inches.

Diameter of capital $= \frac{L}{4 \cdot 44} = 4$ feet 6 inches.

Width of drop panel $= \frac{L}{3} = 6$ feet 8 inches.

Total load, $W = 20 \times 20 \times 250 = 100\ 000$ lb.

Strip A, positive moment (mid-way between columns) =

$$\frac{W\ L}{80} = \frac{100\ 000 \times 20 \times 12}{80} = 300\ 000 \text{ inch-lb.}$$

Strip A, negative moment (over column), =

$$\frac{W\ L}{30} = \frac{100\ 000 \times 20 \times 12}{30} = 800\ 000 \text{ inch-lb.}$$

Strip B, positive moment (center of panel) =

$$\frac{W\ L}{120} = \frac{100\ 000 \times 20 \times 12}{120} = 200\ 000 \text{ inch-lb.}$$

Strip B, negative moment between columns =

$$\frac{W\ L}{120} = 200\ 000 \text{ inch-lb.}$$

Moment for wall column $= \frac{W L}{60} =$ 400 000 inch-lb.

Least full thickness of slab $= \frac{\sqrt{W}}{44} = 7{\cdot}2$ inches.

The thickness of the drop panel must be determined by computing it as a beam, but it must not be less than required to take care of the punching shear around the perimeter of the column capital, using the allowable unit shearing stress of 120 lb. per square inch. The shear area will equal the perimeter of the cap multiplied by the thickness ; this area multiplied by 120 will give the total shear the concrete will take ; therefore, $S = 120\ p\ t$, where $p =$ perimeter of capital, $= 54 \times 3{\cdot}1416$, say 170 inches. S = total shear around one capital, which will equal the total load for one panel less that directly over the column capital, it therefore equals $100\ 000 - 0{\cdot}7854 \times 4{\cdot}5 \times 4{\cdot}5 \times 250 = 96\ 125$ lb.

Then $$t = \frac{S}{120\ p} = \frac{96\ 125}{120 \times 54 \times 3{\cdot}1416} = 4{\cdot}72\ ;$$

which is all that is necessary for the punching shear. Computing the thickness as for a beam we can apply the formula used throughout this work, where $d = \sqrt{\frac{M}{95\ b}}$. For the drop panel, $b = 80$ inches. M, as already found, equals 800 000 inch-lb.

Hence, $$d = \sqrt{\frac{800\ 000}{95 \times 80}} = 10{\cdot}26 \text{ inches,}$$

which is the greatest of the three requirements. $a = p\ b\ d = 0{\cdot}006\ 75 \times 80 \times 10{\cdot}26 = 5{\cdot}55$ square inches.

For strip A, midway between the columns,

$$M = \frac{W L}{80} = 300\ 000 \text{ inch-lb.}\quad d = \sqrt{\frac{300\ 000}{95 \times 10 \times 12}} = 5{\cdot}13,$$

or say a full thickness of 6 inches, the minimum allowed. $a = 0{\cdot}006 \quad 75 \times 120 \times 5{\cdot}13 = 4{\cdot}155$ square inches.

For strip B, negative moment for between columns, and positive moment for center of panel will each equal 200 000 inch-lb.

The effective thickness of the slab is already determined as 5·13 inches ; then a, for both positions, equals

$$\frac{M}{0{\cdot}88\, t\, d} = \frac{200\ 000}{0{\cdot}88 \times 16\ 000 \times 5{\cdot}13} = 2{\cdot}76 \text{ square inches.}$$

The reinforcement for the negative moment over the columns is divided into the cross and diagonal bands. Authorities differ on the proportions into which it should be divided, some take one-third for the diagonal, and two-thirds for the cross band ; others take a proportion between this and an equal division. Tests on completed floors under working conditions tend to show that the stresses in the diagonal and cross bands are approximately equal, whether it is actually so or not, it is evident from the satisfactory performance of existing structures that an equal distribution of this portion of the steel is perfectly satisfactory.

For the case under consideration, making the steel for the cross and diagonal bands equal, the steel in the top of the slab over the column, will be $\frac{5{\cdot}55}{2} = 2{\cdot}775$ square inches, say 12 bars $\frac{1}{2}$ inch square for both the cross band and the diagonal band. The bars are usually made to lap over the column, so this would mean 6 bars from each side, each extending over the column to the quarter point of the span on the other side.

The steel for the bottom of the slab in the center of the panel, i.e., the center portion of strip B, is the amount of steel in a diagonal band.

The steel for the positive moment in each strip A, i.e., in the bottom of the slab midway between the columns, shall be the area of steel in a cross band which equals 4·155 square inches, say 17 rods $\frac{1}{2}$ inch square.

The steel for the negative moment in each strip B, i.e., in the top of the slab midway between columns, equals 2·76 square inches, say 11 rods $\frac{1}{2}$ inch square, or 20 rods $\frac{3}{8}$ inch square.

The width of the cross and diagonal steel bands are usually taken about four-tenths of the width of the panel, in this case say 8 feet; the arrangement of the bands will then be as Fig. 95.

All rods over the columns are to be bent up to near the top of the slab, the bent up portion extending one quarter of the panel length on each side of the column center.

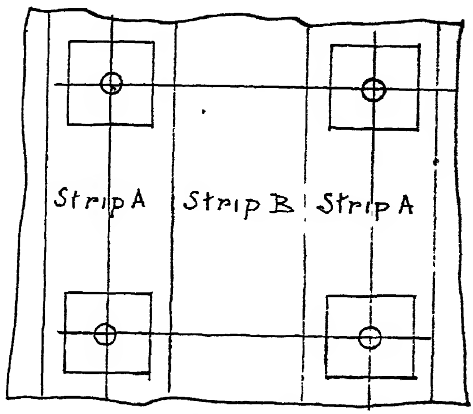


FIG. 94.

Example XXVIII.—By the formulas recommended in the Report of the American Joint Committee, determine the design for a two-way flat slab floor with drop panels, for the same load and column spacing as that of example XXVII.

The panel will be divided as Fig. 94; the formulas to use are as follows:—

Diameter of column capital, $0{\cdot}225\ \mathrm{L} =$ 4 feet 6 inches.

Width of drop panel, $\frac{\mathrm{L}}{2{\cdot}5} =$ 8 feet.

Minimum thickness of drop panel, $0{\cdot}03\ \mathrm{L}\sqrt{w}+1''$, but not less than 6 inches, or $\frac{\mathrm{L}}{32}$; therefore, least thickness equals $0{\cdot}03 \times 20 \times \sqrt{250}+1'' = 10\frac{1}{2}$ inches.

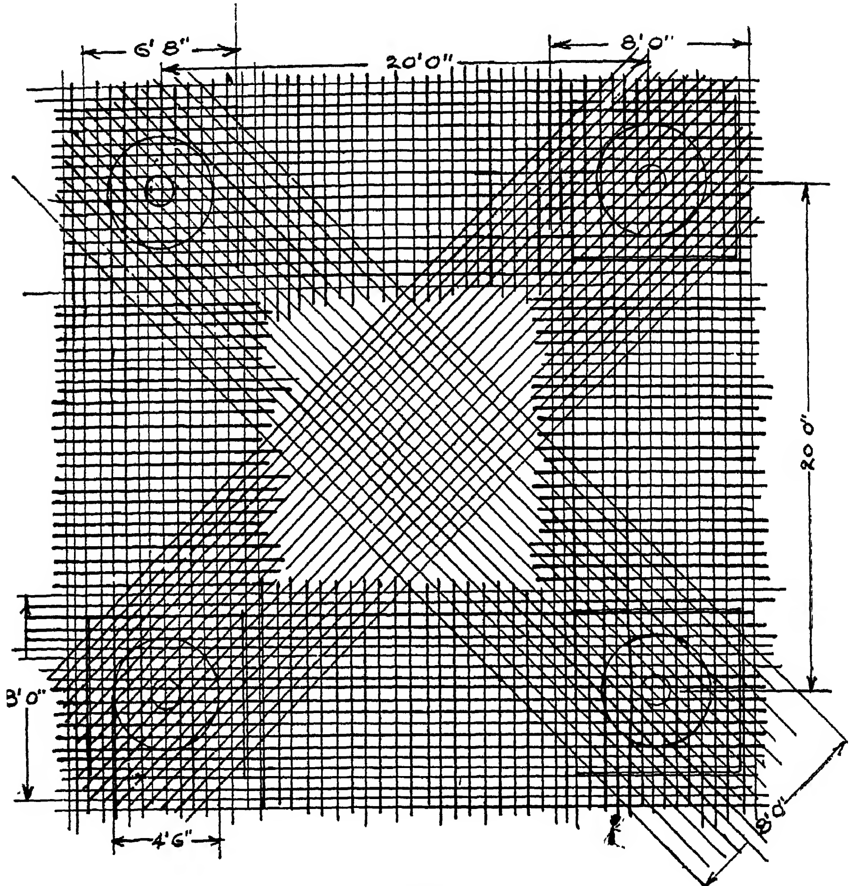


FIG. 95.

Minimum thickness of slab

$$0{\cdot}02\ \mathrm{L}\sqrt{w}+1'' = 0.02\times 20\sqrt{250}+1'' = 7\tfrac{1}{3}\text{ inches.}$$

Positive $\mathrm{M} = \dfrac{\mathrm{W\ L}}{25}$, where $\mathrm{W} = w\left(\mathrm{L}-\dfrac{2}{3}\,c\right)^2$,

and c =diameter of column capital.

$$\text{Therefore}\quad \mathrm{W} = 250\left(20-\frac{2}{3}\times 4{\cdot}5\right)^2 = 72\,250\text{ lb.}$$

$$\text{Then}\ +\mathrm{M} = \frac{72\,250\times 20\times 12}{25} = 693\,600\text{ inch-lb.}$$

Negative M, for over columns,

$$= \frac{W\ L}{15} = \frac{72\ 250 \times 20 \times 12}{15} = 1\ 156\ 000 \text{ inch-lb.}$$

The thickness for the drop panel and the slab can be determined by the formula used throughout this work,

where $$d = \sqrt{\frac{M}{95\ b}}.$$

For the thickness of the slab away from the drop panel the recommendations are that 70 per cent. of the positive moment should be considered as acting in the two outer sections, which two sections are equivalent to a portion between the columns equal in width to half the panel. Then for the thickness of slab the moment will be 70 per cent. of 693 600 = 485 520 inch-lb.

Hence, $$d = \sqrt{\frac{485\ 520}{95 \times 10 \times 12}} = 6{\cdot}5 \text{ inches,}$$

or a full thickness of $7\frac{1}{2}$ inches, which agrees with the minimum requirements.

It is also recommended that at least 25 per cent. of the positive moment should be provided for in the inner section, or across the center portion of the panel, and that at least 60 per cent. should be provided for across the center of the two outer panels, that is across a portion midway between the columns equal in width to half of the whole panel. According to this, the moment for the inner section need not be taken at more than $\frac{25}{70}$, about 36 per cent., of that used for determining the thickness of the slab ; consequently, to provide for this amount only, less steel can be used than would be given by the formula, $a = p\ b\ d$, for if this formula were used there would be sufficient steel to provide for the 70 per cent. moment used for the slab thickness, instead of the 25 per cent. This is therefore a case in which we have more concrete than is necessary for what we call the

economic section, and is a case for which we can determine the steel by the formula

$$a = \frac{M}{0{\cdot}88\, t\, d}.$$

Hence, the moment for the steel equals

$$\frac{25}{100} \times 624\ 240 = 156\ 060 \quad \text{inch-lb.}$$

Then $a = \dfrac{156\ 060}{0{\cdot}88 \times 16\ 000 \times 6{\cdot}2} = 1{\cdot}788$ square inches,

for the whole width of 10 feet.

Across the portion midway between the columns 60 per cent. of the positive moment should be provided for;

therefore, $a = \dfrac{60 \times 624\ 240}{100 \times 0{\cdot}88 \times 16\ 000 \times 6{\cdot}2} = 4{\cdot}29$ square

inches per foot of width.

For the drop panels the recommendations are that at least 80 per cent. of the negative moment should be provided for; then the moment here will be $0{\cdot}8 \times 1\ 156\ 000 = 924\ 800$ inch-lb., and b for the drop panel equals 8 feet; therefore,

$$d = \sqrt{\frac{924\ 800}{95 \times 8 \times 12}},$$

say 10 inches, or a full thickness of about 11 inches, which also agrees with the minimum requirements.

As the moment to be provided for is the same as used to determine the thickness of the concrete the formula, $a = p\ b\ d$, can be used for the reinforcement; hence, using $\frac{5}{8}$ inch square bars, the section of one will contain 0·3906 square inches. The distance apart will therefore equal

$$b = \frac{a}{p\ d} = \frac{0{\cdot}3906}{0{\cdot}006\ 75 \times 10} \text{ say } 5\tfrac{3}{4} \text{ inches.}$$

For all panels that are not continuous, such as those that are next to the walls, or against openings, the reinforcement in the bottom of the slab midway from the wall, or opening, to the first line of columns, in both strips A and B, must be increased 25 per cent. to take care of the extra bending moment at these points.

Example XXIX.—Determine the thickness of the slab, and the reinforcement for a two-way flat slab floor panel to comply with the requirements of the Chicago and Los Angeles building codes. The span and load to be the same as in the previous example.

Diameter of column equals $\frac{20 \times 12}{12} = 20$ inches.

Diameter of column capital $= 0{\cdot}225 \times 20 = 4$ feet 6 inches.

Width of drop panel, not less than one-third of span $= \frac{20}{3} = 6$ feet 8 inches.

Negative moment over column capital $= \frac{W\ L}{30}$ where W equals load on whole panel, $= 20 \times 20 \times 250 = 100\ 000$.

$$\text{Then } M = \frac{100\ 000 \times 20 \times 12}{30} = 800\ 000 \text{ inch-lb.}$$

The thickness of the slab and the reinforcement can be determined by the formulas used throughout this work ; hence, thickness of drop panel

$$= \sqrt{\frac{M}{95\ b}},$$

where b equals width of drop panel ; therefore

$$d = \sqrt{\frac{800\ 000}{95 \times 80}} = 10{\cdot}25 \text{ inches.}$$

Allowing 1 inch for cover the full thickness will be $11\frac{1}{4}$ inches. The required reinforcement equals $p\,b\,d =$ $0{\cdot}006\ 75 \times 80 \times 10{\cdot}25 = 5{\cdot}535$ square inches, say $\frac{5}{8}$ inch square rods at $5\frac{1}{2}$ inch centers for the full width of strip A.

The positive bending moment for center of strip A $= \dfrac{W\ L}{60}$, half that over the columns, or 400 000 inch-lb.

Thickness of slab

$$= \sqrt{\frac{400\ 000}{95 \times 10 \times 12}} = 5{\cdot}92 \text{ inches.}$$

The full thickness, according to the ordinance, must not be less than

$$\frac{\sqrt{w}}{44} = \frac{\sqrt{100\ 000}}{44} = 7{\cdot}2 \text{ inches,}$$

nor must it be less than

$$\frac{L}{32} = \frac{20 \times 12}{32} = 7{\cdot}5 \text{ inches.}$$

The latter, being the greatest, determines the thickness. Allowing 1 inch cover, the effective depth will therefore be $6\frac{1}{2}$ inches, instead of $5{\cdot}92$, as found to be all that is really necessary.

The effective depth being more than is necessary for the economic section the reinforcement can be found by the formula,

$$a = \frac{M}{0{\cdot}88\ t\ d} = \frac{400\ 000}{0{\cdot}88 \times 16\ 000 \times 6{\cdot}5} = 4{\cdot}37 \text{ square inches.}$$

The positive bending moment for the center of strip B, in the middle of the panel, and the negative bending moment for the center of the same strip, on the center line of columns, are to be taken as the same, and will equal $\dfrac{W\ L}{120}$, or half that taken for strip A, midway be-

tween column centers. It therefore equals 200 000 inch-lb. The thickness of the slab is already fixed at $6\frac{1}{2}$ inches, we therefore use this to determine the reinforcement for these center portions.

$$\text{Then } a = \frac{200\ 000}{0{\cdot}88 \times 16\ 000 \times 6{\cdot}5} = 2{\cdot}19 \text{ square inches,}$$

or half that required for the center of strip A.

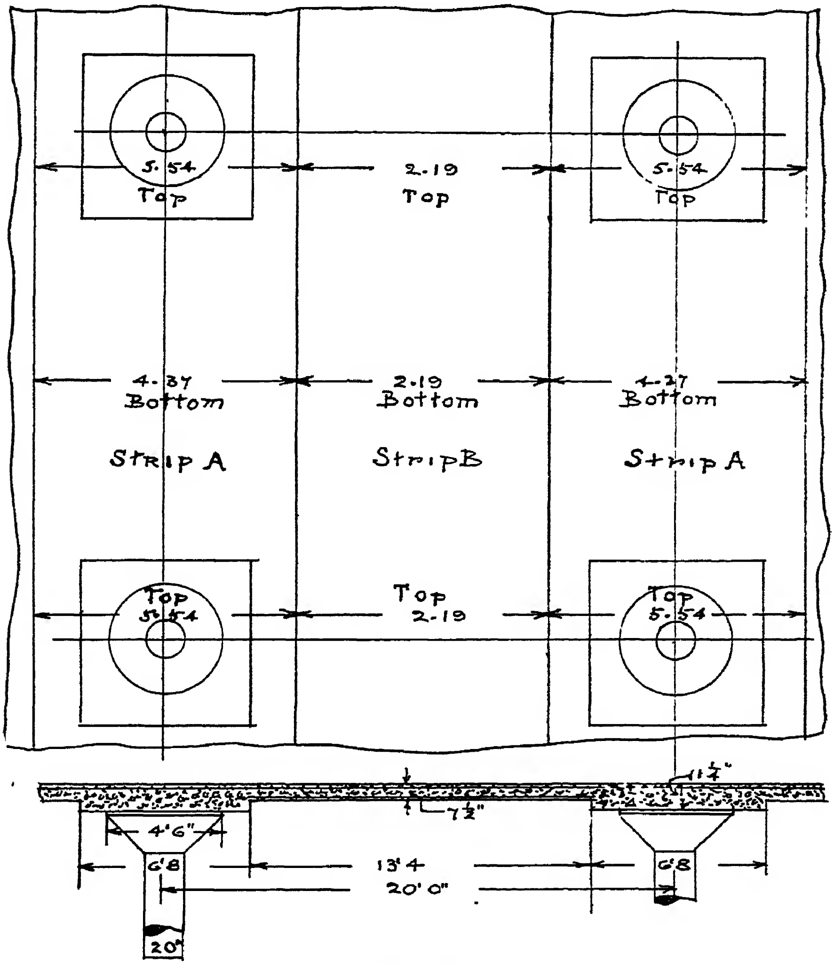


FIG. 96.

We now have the following data :—

Diameter of column	20 inches
Diameter of column capital	4 feet 6 inches
Width of drop panel	6 feet 8 inches
Thickness of drop panel	$11\frac{1}{4}$ inches
Thickness of slab	$7\frac{1}{2}$ inches
Reinforcement in Strip A, in top of slab, over columns	5 535 square inches
Reinforcement in Strip A, in bottom of slab, centre portion between column	4 37 square inches
Reinforcement in Strip B, in bottom of slab at middle of panel	2.19 square inches
Reinforcement in Strip B, in top of slab on centre line of Column	2 19 square inches

For the floor panels that are not continuous, such as those adjoining the outer walls, the reinforcement in the bottom of the slab midway between the wall and first line of columns, in both strips, A and B, must be increased to take care of the 25 per cent. increase in the bending moment of these points. It will therefore require to be 5·46 and 2·74 square inches respectively at these points.

The plan and section, through the drop panels, will be as Fig. 96. The figures on the plan being the steel required in the widths indicated.

SHEARING REINFORCEMENTS.

When the shear stress on the concrete exceeds the safe limit, special shear members must be used. When these are required some engineers design them to take all the shear ; this, however, is not necessary, as the introduction of additional steel does not diminish the shear strength of the concrete., The shear stress is sometimes called diagonal tension, but it should not be, as there is diagonal tension, also diagonal compression as well as shear ; the tension, compression, horizontal shear and vertical shear are of equal intensity at any point. The compression and tension act at right angles to one another, and at 45 degrees to the shear, as shown in Fig. 97. The shear creates a tendency for the con-

crete to fracture along the lines S—S and the tension tends to pull the concrete apart in the direction of the compression line C; consequently, reinforcement placed at 45 degrees in the direction of the line T, will be in tension providing the adhesion of the concrete to the steel is sufficient to allow the tension to act on the steel.

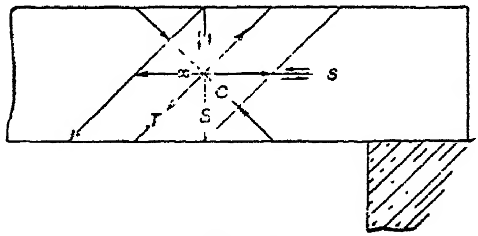


FIG. 97.

For any distance, as x, Fig. 97, the horizontal shear will equal s multiplied by the distance x, where s equals the shear per inch of length for the steel. If the reinforcement is to be placed vertical sufficient must be employed to take this amount; therefore, the sectional area of steel required will equal $\frac{s\,x}{t}$, where t = the safe shear per square inch for the steel, usually 12 000 lb.

If the reinforcement is placed along the line T, it will be in tension, and the amount it will have to resist for the horizontal length x will be that acting along a distance equal to c, which distance is to x as 1 is to the $\sqrt{2}$; consequently, reinforcement inclined at 45 degrees will be stressed equal to $\frac{1}{\sqrt{2}}$ times that of vertical reinforcement, and the stress will be tension so can be taken at 16 000 lb. per square inch, instead of 12 000 lb. per square inch. It is therefore economical to employ inclined shear members, for although they are equal in length to the $\sqrt{2}$ times the vertical members the reduction in sectional area will more than compensate for the additional length.

As the sectional area required for vertical reinforcement equals $\frac{s\,x}{t}$ that for inclined reinforcement, on the

above reasoning, will equal $\frac{1}{\sqrt{2}}$ times $\frac{s\,x}{t} = \frac{s\,x}{\sqrt{2}\,t}$, where t = the limiting tension for the steel.

Whether to adopt vertical or inclined members will depend to a certain extent on the disposition of the longitudinal stresses and bars.

In a beam supported at the ends where the bending moment diminishes towards the supports, it is possible to crank up some of the tension bars, at a point where the bending moment is sufficiently reduced, and run them along to take the vertical shear, the most effective position being along, or just below, the neutral axis. Vertical members are then required to take the horizontal shear.

When the longitudinal bars cannot be used for this purpose, inclined members must be adopted.

Vertical and inclined members are arranged in the form of stirrups, and should be firmly clipped, or wired, to the longitudinal bars, and further anchored by hooking their top ends. The distance from center to center of the stirrups should not exceed the effective depth of the beam, and they should extend from the tensile reinforcement to at least one-third n from the compression surface.

The American Joint Committee on Reinforced Concrete, recommend that the spacing of vertical stirrups should not exceed half the depth of the beam, and that for inclined stirrups three-quarters the depth of the beam; others recommend two-thirds the depth for either vertical or inclined stirrups.

The whole of the concrete above the tension bars assist in resisting the shear; the stress, however, is not uniformly distributed over this area, but is greatest where the longitudinal stresses are least, i.e., at and below the neutral axis. Above the neutral axis it diminishes, as ordinates of a parabola, to zero at the surface. The distribution below the neutral axis is rather uncertain, for if the concrete should be acting in resistance to tension, the shear will diminish in a similar

manner to that above the axis, but when a beam is fully stressed and the usual assumption is correct, that there is no tension in the concrete, the shear below the axis will be uniformly distributed. On this assumption the distribution of shear across the section will be as Fig. 98.

As the area of a parabola equals two-thirds of a rectangle of the same width and height, the total shear above the axis will equal $\frac{2}{3} b\,n\,s$, where $s =$ the shear per square inch. The shear below the axis will equal $b\,(d - n)\,s$; hence the total shear throughout the section will be, $S = \frac{2}{3} b\,n\,s + b\,(d - n)\,s = s\,b\left(d - \frac{n}{3}\right)$. From this, s, or the shear per square inch will be $\dfrac{S}{b\left(d - \frac{n}{3}\right)}$, and the shear per inch of depth below the axis will be $\dfrac{S}{d - \frac{n}{3}}$.

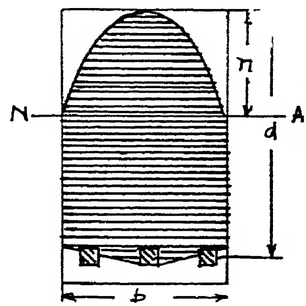


FIG. 98.

In slabs and rectangular beams with single reinforcement, designed to enable both the concrete and steel to be stressed to their allowable limits, as explained in the foregoing examples, there is seldom need for special shear members as the section generally turns out sufficient in area to give the necessary resistance. Although this is the case it is often advisable to use them, if they are required or not will depend on the purpose for which the structure is designed. If for a structure that may have to resist fire, or if it will be exposed to sudden and extreme changes of temperature, shear members will assist in holding the concrete together when expanding or contracting; for this purpose they should be small and numerous rather than large and scarce.

ADHESIVE STRESS.

The intensity of shear around the bars is known as the adhesive or bond stress, it will equal the shearing stress across the section at the depth of the bars $= \dfrac{S}{d - \frac{n}{3}}$. This value divided by the perimeter of the bars will give the adhesive stress per square inch of bar surface. If this stress is found to exceed the safe allowance, usually 100 lb. per square inch, further resistance can be obtained by securely fixing the shear members to the bars.

Indented, twisted, or other deformed bars are often used for the purpose of increasing the adhesive resistance; extensive tests and experiments, however, indicate that there is seldom, if ever, need to use any other than a plain bar to resist this stress, as these are capable of resisting far more than they are usually called upon to develop. A plain round bar with ordinary mill surface will resist from 300 to 400 lb. per square inch of surface before the adhesive resistance is overcome and slip occurs. Bars with a fine rust coating will resist up to 15 per cent. more than mill surface bars, and polished bars about 60 per cent. less than mill surface bars. The sliding resistance of any bar is due to inequalities of the surface of the bar and to irregularities of its section and alignment. The projections on a deformed bar give an exaggerated condition of inequality of surface or irregularity of section. Adhesive resistance must be destroyed and sliding resistance largely overcome, and the concrete in front of the projections must undergo an appreciable compressive deformation, before the projections on a deformed bar become effective in taking bond stress. Tests indicate that the projections do not materially assist in resisting a force tending to withdraw the bar until a slip has occurred approximating that corresponding to the maximum sliding resistance of a plain bar; consequently, there appears to be no advantage in the use of deformed bars over plain bars to take care of the

adhesive stresses. They certainly offer greater resistance to sliding after the adhesion is overcome, but the value of reinforced concrete is largely due to the concrete and steel being so combined as to act as one, which they cannot do if the adhesion of the concrete to the steel is not perfect.

The length of any rod embedded in concrete must be such as to permit of sufficient grip or adhesion to resist the direct stress of compression or tension in the rod, otherwise the concrete and steel will not act together, and the rod will slip before the full stress can take effect. For example, take a rod $\frac{1}{2}$ inch square to be stressed to 16 000 lb. per square inch. The stress in the rod will equal its sectional area multiplied by $16\,000 = 0{\cdot}25 \times 16\,000 = 4\,000$ lb. Allowing 100 lb. adhesion per square inch of surface, the total adhesion will be 100 times the perimeter of the rod multiplied by the length, which must equal 4 000. The perimeter $= 4 \times 0{\cdot}5 = 2$ inches;

hence, $4\,000 = 2 \times 100\, l$, then $l = \dfrac{4\,000}{200} = 20$ inches.

From this reasoning the adhesion length equals $\dfrac{t\,a}{O \times 100}$ where $O =$ perimeter of rod. For a square rod $a =$ dia.^{2}, and the perimeter $= 4$ times dia.,

therefore, $$l = \frac{t\,d^2}{100 \times 4\,d} = \frac{t\,d}{400},$$

For a round rod

$$l = \frac{t\,\dfrac{\pi}{4}\,d^2}{100\,\pi\,d} = \frac{t\,d}{400}.$$

The same rule is therefore, applicable to both square and round rods, and whether in compression or tension. From this rule it will be seen that the adhesion length must not be less than 40 diameters for 16 000 lb.; 35

diameters for 14 000 lb.; and 30 diameters for 12 000 lb. per square inch stress.

Take the following as a further example. The rods in a column are $\frac{3}{4}$ inch, and are to be lengthened by lapping and stressed to 10 000 lb. per square inch. For this the length of lap should be not less than

$$\frac{10\,000 \times \frac{3}{4}}{400} = 19 \text{ inches.}$$

If the rods are square and close together, there will be only three sides of each to which the concrete can thoroughly adhere, then the lap length should be not less than $\frac{10\,000 \times \frac{3}{4}}{300} = 25$ inches.

For rectangular rods the adhesion length must be not less than $\frac{t\,a}{200\,(b+d)}$ where b and d = the width and thickness of the rod.

In the following examples of the use of shear formulas the beams of the previous examples are taken to complete their design.

Example XXX.—Taking the beam of Example VII., the effective depth is given as 17·4 inches, b as 10·5 inches, and a equals 3 bars $\frac{11}{16}$ inch square.

$$S = \text{the greatest shear} = \frac{W}{2} \text{ at the supports} = \frac{500 \times 20}{2}$$

= 5 000 lb. The greatest shear on the concrete, per square inch, equals

$$s = \frac{S}{b\left(d - \frac{n}{3}\right)}, \text{ and } n = 0{\cdot}36\ d = 0{\cdot}36 \times 17{\cdot}4 = 6{\cdot}264.$$

Hence, $$s = \frac{5\,000}{10{\cdot}5 \times 15{\cdot}4} = 31 \text{ lb.;}$$

which is only 50 per cent. of the allowable stress. The intensity of shear per square inch around the reinforcing bars will equal $\dfrac{S}{O\left(d-\frac{n}{3}\right)}$.

Where O = the perimeter of the bars,

$$= \frac{5\ 000}{\frac{11}{16} \times 4 \times 3 \times 15{\cdot}4} = 39 \text{ lb.},$$

which is much within the safe limit.

Example XXXI.—Taking the beam of Example VIII.

$$s = \frac{S}{b\left(d-\frac{n}{3}\right)} \text{ and } S = \frac{W}{2},$$

which, by taking the actual weight of the designed beam, will be $\dfrac{28\ 850}{2} = 14\ 425$ lb.

$d = 26$ inches, $b = 15\frac{1}{2}$ inches, $a = 4$ bars $\frac{7}{8}$ inch square, and $n = 0{\cdot}36\ d = 9{\cdot}29$ inches.

Therefore $\quad s = 14{\cdot}5 \dfrac{14\ 425}{\left(25{\cdot}8 - \frac{9{\cdot}29}{3}\right)} = 41$ lb.

The adhesive stress around the bars will be

$$\frac{S}{O\left(d-\frac{n}{3}\right)} = \frac{14\ 425}{\frac{7}{8} \times 4 \times 4 \times 22{\cdot}7} = 45 \text{ lb. per square inch.}$$

In this case also the shear is much below the safe limit.

Example XXXII.—Taking the beam of Example XXV. The effective depth equals 13·76 inches; the breadth equals 13 inches, and $a =$ 5 bars $1\frac{1}{4}$ inch square.

As the slabs do not receive any support directly from the side walls, i.e., the walls that support the beams, they must not be taken with the beams as resisting the shear at the supports; consequently, the shear will be as for a rectangular beam, where

$$s = \frac{S}{b\left(d - \frac{n}{3}\right)}$$

Whether the slabs receive support from the walls or not it is the general practice to ignore their resistance and consider the beams as resisting all the shear.
The shear at the supports equals

$$S = \frac{W}{2} = 24\ 000 \text{ lb.},$$

and $$n = 0{\cdot}36\ d, = 4{\cdot}95 \text{ inches};$$

hence $$s = \frac{24\ 000}{13 \times 12{\cdot}11} = 152 \text{ lb.}$$

As this is more than twice as much as the concrete can take, it is therefore evident that special shear members must be used.

Allowing for the maximum shear to be 60 lb. per square inch, the total shear for the concrete across the section will be

$$60\ b\left(d - \frac{n}{3}\right) = 60 \times 13 \times 12{\cdot}11 = 9\ 446 \text{ lb.}$$

This leaves a balance at the supports for shear members of $24\ 000 - 9\ 446 = 14\ 554$ lb.

The safe resistance of the steel to shear may be taken

as 12 000 lb. per square inch ; hence the area required will be $\frac{14\ 554}{12\ 000} = 1{\cdot}21$ square inch. This can be provided by cranking up one of the tension bars and running it along at half the depth. It can be bent up from a point where the shear is reduced to the amount that the concrete will resist, providing the bending moment at this point is sufficiently reduced to admit of this being done.

In a beam with a uniformly distributed load the shear diminishes uniformly from the supports to zero at the center, the stress at any point along the length being equal to the load between the point and the center of the beam, or we may say, equal to the reaction of the nearest support minus the load on the beam between that support and the point under consideration.

The load per lineal foot equals $\frac{48\ 000}{20} = 2\ 400$ lb.

The shear allotted to the concrete is 9 446 lb. ; therefore, the distance from the center of the beam to where the bar must be cranked up will be $\frac{9\ 446}{2\ 400}$, say 4 feet, as shown in Fig. 99.

The tendency for the concrete to shear along a horizontal plane, which at any point is equal to the vertical shear at the same point, may be resisted by vertical members. It is usual to make these in the form of stirrups, equal in size, but spaced so as to be equally stressed. To do this it is best to find the total area of steel required to resist the whole of the excess stress along a plane through the neutral axis ; as the shear here is greatest there will then be sufficient steel to resist the stress at any other point of the depth ; the amount can then be divided into stirrups of suitable size, and placed at distances which will allow them to be equally stressed. Taking one-half the length of the beam, and allowing for the concrete to take the same amount as for the vertical shear, also neglecting the small resistance

of the slab, which is uncertain and in any case will tend towards safety; the total shear to be provided for can be determined as follows:—

We have found the stress for the shear members at the supports to be 14 554 lb., therefore $\dfrac{14\,554}{d - \frac{n}{3}}$ equals the shear stress across the breadth at the neutral axis $= \dfrac{14\,554}{12{\cdot}11}$, say 1 200 lb. We have also found that the concrete is capable of resisting all the shear at 4 feet

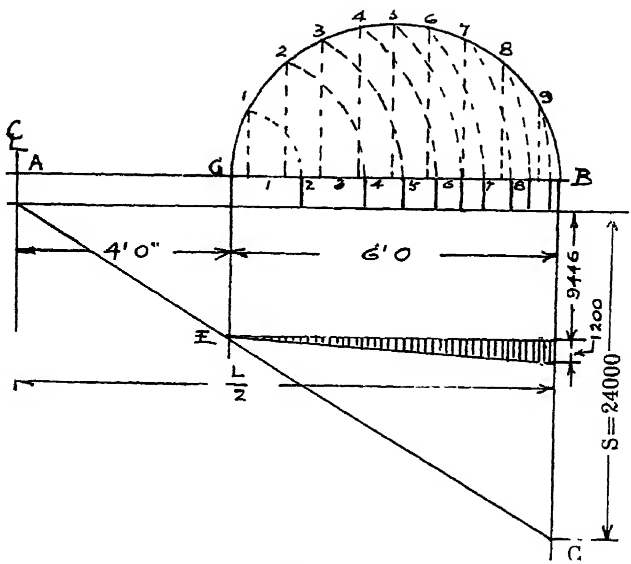


FIG. 99.

from the center of the beam. Now, as the shear diminishes from the supports to zero at the center as ordinates of a triangle, the excess for which we require reinforcement will diminish to zero at the point where the concrete is capable of resisting the whole stress; consequently, the total excess along the neutral axis will equal one-half of the excess at this depth, at the support

multiplied by half the length of the beam minus 4 feet, thus :—

$$\frac{1\ 200\ (10 \times 12 - 48 \text{ inches})}{2} = 43\ 200 \text{ lb.}$$

This apportionment and diminishing of the shear stress is illustrated by the diagram, Fig. 99, where the triangle A B C represents the total shear for one-half the beam ; by scaling off from B to D the amount allotted to the concrete, the line D E drawn parallel to A B will intersect A C at a point where the excess of stress diminishes to zero. Now by scaling off D F equal to the proportion of the stress for the reinforcement at the neutral axis, i.e., 1 200 lb., the area of the triangle D E F will represent the total shear along a plane through, or below, the neutral axis and equals the amount for which the stirrups must be designed, it will therefore be

$$\frac{\text{D F} \times \text{D E}}{2} = \frac{1\ 200 \times 6 \times 12}{2} = 43\ 200 \text{ inch-lb.}$$

Allowing 12 000 lb. per square inch as a safe shear stress for the steel, the total area required will be $\frac{43\ 200}{12\ 000} = 3 \cdot 6$ square inches. This amount can be divided into as many parts as we wish to have stirrups. For nine sets of two stirrups each they will each require an area of $\frac{3 \cdot 6}{2 \times 9} = 0 \cdot 2$ square inch for each stirrup. A stirrup having two branches, the section of each branch will be 0·1 square inch, say $\frac{5}{8}$ inch by $\frac{3}{16}$ inch flat bar, or $\frac{1}{2}$ inch by $\frac{1}{4}$ inch if preferred ; no stirrup steel should be less than $\frac{1}{16}$ inch thick. The spacing can be determined as shown by the upper portion of the diagram Fig. 99, which is constructed as follows :—Make B G equal the portion of the beam in which the stirrups are to be placed, describe the semi-circle, and divide B G into as many equal parts as there are to be stirrups ; from the center

of each division, draw perpendicular lines to intersect the semi-circle in points 1, 2, 3, etc., from G with radius to each of the divisions on the semi-circle draw arcs which will intersect G B at points where the stirrups are to be placed.

The complete sections will be as Fig. 78 and 79.

Example XXXIII.—Taking the doubly reinforced beam of Example XXVI. The maximum shear per square inch equals

$$s = \frac{S}{b\left(d - \frac{n}{3}\right)}$$

and $S = \frac{W}{2} = \frac{36\,188 + 26\,812 + 7\,583}{2}$ (see page 159)

$$= 35\,292 \text{ lb}$$

$b =$ 14 inches, $d =$ 24 inches, and $\frac{n}{3} = 2{\cdot}88$.

Therefore $\quad s = \frac{35\,292}{14 \times (24 - 2{\cdot}88)} = 120$ lb.

As this exceeds the safe limit shear members must be used. Allowing the concrete to take 50 lb. per square inch, the total for the concrete will be

$$50\,b\left(d - \frac{n}{3}\right) = 50 \times 14 \times 21{\cdot}12 = 14\,784 \text{ lb.}$$

The balance for shear members at the supports will be $35\,292 - 14\,784 = 20\,508$ lb. At 12 000 lb. per square inch, the sectional area of steel required will be $\frac{20\,508}{12\,000} =$ 1·71 square inch. In this case the longitudinal bars cannot be cranked up, as the same quantity is required for tension and compression at the supports as at the center.

Additional longitudinal bars could be used for the vertical shear, and vertical members designed to take the horizontal shear ; this, however, is not so practicable or economical as using inclined members.

In this case, the shear will not diminish to zero at the center of the beam, as we have here a concentrated load of 36 188 lb., which will cause a shear of $\frac{36\ 188}{2} = 18\ 094$ lb. at any point between the center and the supports. To obtain the total shear for one-half of the beam we must add the shear from the distributed load. On page 158 the distributed load is given as $11\ 250 + 26\ 812 =$ 38 062 lb. From this the shear will be $\frac{38\ 062}{2} = 19\ 031$ lb., this portion, however, will diminish to zero at the center, therefore, the total shear at the supports will equal $18\ 094 + 19\ 031 = 37\ 125$ lb., which diminishes to 18 094 lb. at the center. The shear diagram being as Fig. 100.

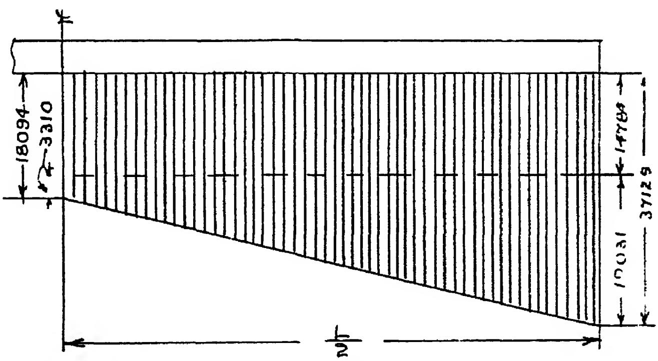


FIG. 100.

The shear allotted to the concrete is 14 784 lb., which will leave a balance at the supports of $37\ 125 - 14\ 784 =$ 22 341 lb., and at the center $18\ 094 - 14\ 784 = 3\ 310$ lb.

The shear for the steel per inch of depth at the supports will be

$$\frac{S}{d - \frac{n}{3}} = \frac{22\ 341}{21 \cdot 12} = 1\ 058 \text{ lb.}$$

At the center $\dfrac{3\ 310}{21 \cdot 12} = 157$ lb.

Then the total excess horizontal shear for half the beam along, or below, the neutral axis will be, the average shear multiplied by half the length of the beam, it therefore equals

$$\left(\frac{1\ 058 + 157}{2}\right) \times 10 \times 12 \text{ inches} = 72\ 900 \text{ lb.},$$

for which stirrups are required.

For vertical stirrups the area of steel required will be

$$\frac{72\ 900}{12\ 000} = 6 \text{ square inches.}$$

For stirrups inclined at 45 degrees the area of steel will be determined as explained on page 189, where it is shown to equal $\dfrac{s\,x}{\sqrt{2}\,t}$ where $s\,x =$ the shear per inch multiplied by the length for which the stirrups are required ; $s\,x$ for this case equals 72 900, hence the area of steel required equals

$$\frac{72\ 900}{\sqrt{2}\,t},$$

which, taking t as 16 000,

$$= \frac{72\ 900}{1 \cdot 414 \times 16\ 000} = 3 \cdot 22 \text{ square inches.}$$

Say 8 sets of two stirrups, out of flat bar $\frac{5}{8}$ inch by $\frac{3}{16}$ inch.

As the shear for the steel does not diminish to zero at any point along the beam, the spacing cannot be determined in exactly the same way as explained for Example

XXXII., which method is only applicable to beams with a uniformly distributed load, or cases where the shear for the reinforcement diminishes to zero at some point between the center and the support.

For the example under consideration, and for similar cases, the spacing can be determined in a somewhat similar manner to that of Fig. 99, as shown by Fig. 101, where A B C D represents the shear for which the stirrups are designed, and A B half the length of the beam.

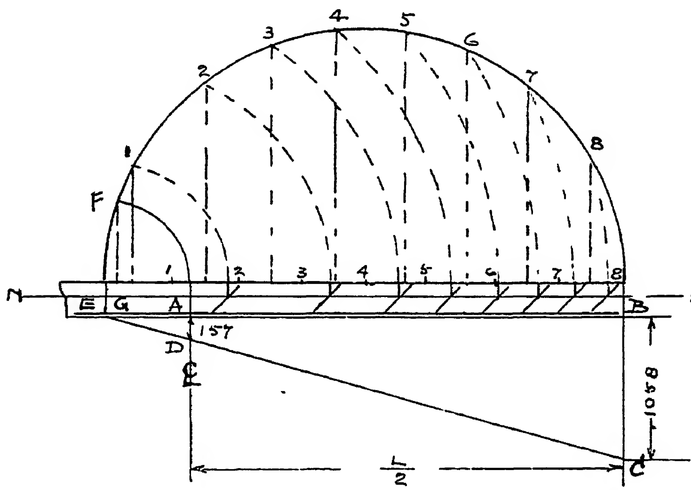


FIG. 101.

Continue C D to E, on B E draw the semi-circle, with radius E A draw the arc A F, from F draw the perpendicular F G, then divide B G into as many parts as there are to be sets of stirrups, and proceed as explained for Fig. 99. The inclined stirrups should be placed so as to pass through the intersection lines and the neutral axis, as shown in Fig. 101. The stirrups must be looped about, and attached to, the horizontal members in such a way as to insure against slip, and should be extended up into the slab with a hook bend at the top.

COLUMNS.

Columns are reinforced with vertical rods tied together at intervals with horizontal bindings of wire, or wire wound in a spiral form around the bars. The object of binding is to prevent lateral expansion of the concrete, buckling of the bars when the column is loaded, and to prevent the bars being bent outwards with the tamping or the weight of the wet concrete.

For a steel bar to act alone as a column and carry its full load without bending, it requires supporting along its length at distances not exceeding 20 times its least diameter, but when the bar is embedded well within the thickness of the concrete stiffeners are not required for this purpose as the concrete prevents lateral movement, but in columns the small thickness of concrete outside the bars offers very little resistance ; owing to this, and to prevent the bars being misplaced by tamping the concrete, binders should be fixed at distances apart not more than 25 times the least diameter of the rods. Some city ordinances call for a spacing of not more than the least dimension of the effective section, nor greater than 15 times the diameter of the longitudinal bars.

When the binding is closer together than six-tenths of the diameter of the column its resistance to the lateral expansion of the concrete adds to the strength of the column, thus enabling the concrete to be stressed considerably higher than would otherwise be possible. The details and strength of this class of column is considered further on.

All binding should be well executed, great care being taken to obtain a tight binding without bending the bars inwards, which bending is almost as faulty as a loose binding. To avoid the possibility of this irregularity, templates should be used to keep the bars in position while the binding is being fixed and the ends securely twisted together.

In columns which are to support live loads, the binding should be closer together near the head to enable the bars to offer greater resistance to the shock without vibration and consequent disturbance of the adhesion bond.

In designing a column, one of the first considerations should be its diameter, which, whenever possible, should be not less than one-fifteenth of its length. The actual strength and the calculated strength can then be relied upon as being practically equal, With a load acting vertically and over the axis of the column the stress is uniform throughout the section, and if it does not exceed 600 lb. per square inch on the concrete, and the column is not greater in length than 15 times its least diameter, there will be no danger of the column failing through bending ; consequently, the load it can support will equal the direct resistance of the concrete, plus the direct resistance of the steel ; the latter, however, cannot be stressed more than m times the stress on the concrete (see page 79) consequently, the total stress will be c multiplied by the sectional area of the concrete, plus m times c multiplied by the sectional area of the steel, where c = the stress per square inch on the concrete and m = the ratio of the co-efficients of elasticity.

The effective diameter of a column is usually taken as the diameter measured from the outside of the vertical reinforcement, which should in no case be less than 7 inches, the concrete outside of this being considered only as a protective covering to the steel. If the covering, however, exceeds $1\frac{1}{2}$ inches the difference can be added to the effective diameter. Some city ordinances call for a cover of at least 2 inches.

Designers vary, from 500 lb. up to 700 lb., in the limiting stress for the concrete in plain columns ; a very satisfactory value is one-fourth of the crushing resistance of 6 inch cubes at two months after mixing.

In rectangular columns there must not be less than 4 vertical rods and in circular columns not less than 6 vertical rods. No rods should be less than $\frac{5}{8}$ inch or more than 2 inches in diameter, and the total reinforcement should be not less than 0·5 per cent., or more than 8 per cent. of the sectional area of the column. Some city ordinances call for not less than 1 per cent, or more than 5 per cent. The American Joint Committee on Reinforced Concrete recommend not less than 1 per

cent. or more than 4 per cent., with lateral ties of steel not less than $\frac{1}{4}$ inch diameter placed 12 inches apart, or not more than 16 diameters of the longitudinal bars, and that the overall dimensions of the columns be not less than 12 inch.

When the vertical rods are not continuous, the joints must be made by lapping, or by dowels or splice rods of equal area to the vertical rods to be supplied. The laps, dowels or splice rods must be of sufficient length to properly transfer the stress from the upper to the lower longitudinal rods, the length can be determined as explained in the chapter on adhesive stress.

Some engineers specify the joints to be made with a tight fitting pipe sleeve, the ends of the connecting rods being milled and close butted ; this method, however, cannot be recommended for eccentrically loaded columns or columns subject to side thrust. In all cases splices should only occur at or near floor levels or points of lateral support.

All the longitudinal rods should extend into the footings or other supports sufficiently far to develop the stress in the rods through adhesion, or dowels may be used instead of the main rods, the length being calculated as for adhesive stress. Dowels are generally used for footings as these can be inserted when the slab is poured and left protruding for the column connection ; the length in the slab and that protruding must each be sufficiently long to meet the adhesive stress requirements.

Example XXXIV.—Determine the load for a circular column 12 inches effective diameter, 10 feet long, reinforced with six round rods 1 inch diameter. The stress on the concrete not to exceed 500 lb. per square inch.

Area of rods equals $6 \times 0{\cdot}785\,4 = 4{\cdot}7$ square inches.

Area of concrete $= 12 \times 12 \times 0{\cdot}785\,4 - 4{\cdot}7$ area of rods $= 108{\cdot}4$ square inches.

Taking $m = 15$, then the load, W, for the column equals $108{\cdot}4 \times 500$ lb. $+ 4{\cdot}7 \times 15 \times 500$ lb. $=$ 89 450 lb.

Instead of finding the net area of the concrete by deducting the area of the steel, a more convenient and shorter method is to assume we have a section wholly of concrete equivalent in strength to the existing section of concrete and steel, by replacing the steel with 15 times its area of concrete.

This equivalent section will equal the full area of the column plus 14 times the area of the bars ; we take 14 times because we have already included the area of the bars once by taking the full section of the column and not deducting the area occupied by the bars. On this assumption we obtain the following equation :—$W = c\,(A + 14\,A_c)$, where W = the load for the column, A = area of the column. A_c = area of the steel, and c = safe stress per square inch for the concrete.

Applying the above equation to the present case we get :—$W = 500\,(113{\cdot}1 + 14 \times 4{\cdot}7) = 89\,450$ lb.

The length of the column does not affect the load unless it is sufficiently long to be liable to bend when loaded, for which cases see next chapter.

Example XXXV. Determine the diameter of a square column to support a load of 200 000 lb., the reinforcement to consist of 8 rods 1 inch square.

By the formula in the previous example, $W = c(A + 14\,A_c)$, from this we get :—

$$A = \frac{W - 14\,A_c\,c}{c},$$

with $c = 500$, and $A_c = 8$ square inches,

$$A = \frac{200\,000 - 14 \times 8 \times 500}{500} = 288 \text{ square inches.}$$

Then for a square column the effective diameter equals $\sqrt{288}$, say 17 inches. Allowing 2 inches for cover, the full diameter will be 21 inches.

Example XXXVI.—Determine the diameter and the reinforcement for a square column to support a load of 250 000 lb. The reinforcement to be 4 per cent. of the effective section of the column.

In the formula, $W = c\,(A + 14\,A_c)$, we have two missing quantities, A and A_c, but A_c is to be 0·04 of A, we can therefore substitute this value for A_c, the formula then becomes :—$W = c\,(A + 14 \times 0{\cdot}04\,A)$, or $c\,A\,(1 + 14 \times 0{\cdot}04)$. From this we get :—

$$A = \frac{W}{c\,(1 + 14 \times 0{\cdot}04)} = \frac{250\,000}{500 \times 1{\cdot}56} = 320 \text{ square inches.}$$

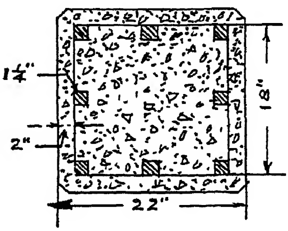


FIG. 102.

Say 8 rods $1\frac{1}{4}$ inch square, giving 12·5 square inches, which is near enough. The effective diameter of the column will be $\sqrt{320} = 17{\cdot}89$ inches. Allowing 2 inches cover the full diameter will be, say 22 inches. The section will be as Fig. 102.

LONG COLUMNS.

With a long column, that is, where the length exceeds 15 times its least diameter, there is a tendency for the column to fail by bending as well as by crushing, consequently it is not so strong as a short column of the same section, therefore the load should be less ; no column should be longer than 30 times its least diameter.

Owing to the lack of experiments with long reinforced columns the exact reduction of the load on what it would be for a short column is at present an uncertain quantity. Some authorities limit the permissible working stress for columns between 15 and 30 diameters long by the following formula :—

$$c' = c\left[1{\cdot}6 - \left(\frac{l}{25\,d}\right)\right].$$

In which c' = permissible working stress for the long

column. c = permissible working stress for the column when $\frac{l}{d}$ is less than 15. l = unsupported length of column, d = effective diameter of column.

Others reduce the load or limiting stress according to their own judgment or theory; an application of Gordon's formula, however, can be relied upon to give a satisfactory reduction, where W = the load for a short column divided by $1+\frac{k\,l^2}{ud^2}$, in which k = a constant depending upon the method of fixing; it equals 1 for both ends fixed, 2·5 for one end fixed and one end rounded or pivoted, 4 for both ends rounded or pivoted.

u = a constant depending upon the shape of the cross section, it is taken as 2 500 for both rectangular and circular solid sections. l = the length of the column, and d = the least diameter of the column.

Applying the first method to the column of the last example, taking the length to be 20 times its effective diameter, say 30 feet,

$$c' = 500\left[1{\cdot}6 - \left(\frac{30 \times 12}{18 \times 25}\right)\right] = 400 \text{ lb.},$$

instead of 500 as allowed for in the given example; a reduction of 20 per cent. for an extension of $33\frac{1}{3}$ per cent. in the length of the column. Then for this length the column would carry 200 000 lb. instead of 250 000 lb. if under 15 diameters long.

By the second method, assuming both ends fixed, the permissible load would be

$$\frac{250\ 000}{1+\dfrac{1 \times 30 \times 30}{2\ 500 \times 1{\cdot}5^2}} = 215\ 517 \text{ lb.}$$

Example XXXVII.—Determine the load for a column 20 feet high, 14 inches square, fixed both ends and reinforced with 4 rods $1\frac{1}{4}$ inch square.

The load for a column less than 15 diameters high

equals c (A+14 A_c). As this column is more than 15 diameters high, the load for a short column of this section, applying Gordon's Formula, must be divided by

the co-efficient $1 + \dfrac{k\ l^2}{u\ d^2}$.

$$\text{Hence W} = \frac{c\ (A+14\ A_c)}{1+\dfrac{k\ l^2}{u\ d^2}} = \frac{500\ (14^2+14\times 4\times 1{\cdot}25^2)}{1+\dfrac{1\times 240^2}{2\ 500\times 14^2}}$$

$$= \frac{141\ 750}{1{\cdot}117} = 126\ 840 \text{ lb.}$$

Working this case with a reduction of the permissible stress by the formula,

$$c' = c\left[1{\cdot}6 - \frac{l}{25d}\right],$$

proceed as follows :

$$c' = 500\left[1{\cdot}6 - \frac{20\times 12}{25\times 14}\right] = 457 \text{ lb.}$$

Then W = 457 (14^2+14 × 6·25) = 129 560 lb.

A difference in the two methods of 2 720 lb.

Example XXXVIII.—Design a column with foundations to support a load 100 000 lb. The height above the foundation slab to be 15 feet, and the soil capable of supporting 4 000 lb. per square foot.

This case may be worked in two ways : (1) By taking a trial section and finding the necessary reinforcement. (2) By deciding on a suitable percentage of reinforcement and finding the section of both concrete and steel.

By No. 1.—Assuming circumstances will admit of the effective diameter being at least one-fifteenth of its

length, we may take it as 12 inches square; allowing $1\frac{1}{2}$ inch cover the full diameter will be 15 inches.

The column is to carry 100 000 lb., so to this must be added the weight of the column itself, which will be $1\frac{1}{4} \times 1\frac{1}{4} \times 15 \times 150$ lb. = 3 516 lb. Therefore, the total load to be carried will be 3 516 + 100 000 = 103 516 lb.

By the formula given in Example XXXIV.,

$$W = c\,(A + 14\,A_c) \text{ or } A\,c + 14\,A_c\;c.$$

Then
$$W - A\,c = 14\,A_c\,c.$$

Hence
$$A_c = \frac{W - A_c}{14\,c} = \frac{103\,516 - 12 \times 12 \times 500}{14 \times 500},$$

say $4\frac{1}{2}$ square inches, 4 bars $1\frac{1}{8}$ inch square, or 8 bars $\frac{3}{4}$ inch square.

If the column is less in diameter than one-fifteenth of its length, Gordon's formula (see page 209) should be incorporated, the equation then becomes :—

$$A_c = \frac{W\left(1 + \dfrac{k\;l^2}{u\;d^2}\right) - A\;c}{14\;c}$$

By method 2, the sectional area of steel may be taken from 0·5 per cent. to 8 per cent. of the sectional area of the column. We then get p A to replace A_c in the equation, where p equals the ratio of steel to concrete. The equation then becomes :—

$$W = A\;c + 14\;p\;A\;c, \text{ or } A\;c\;(1 + 14\;p).$$

In this case, taking p as 5 per cent., we get :—W, including weight of column, $= A\;500\;(1 + 14 \times 0{\cdot}05)$,

$$\text{therefore } A = \frac{103\,516}{500\;(1 + 14 \times 0{\cdot}05)} = 122 \text{ square inches.}$$

Then the side of a square column will be $\sqrt{122}$,

say 11 inches. A_c = 5 per cent. of A, $=0{\cdot}05 \times 122 = 6{\cdot}1$ square inches.

The size of the foundation slab to cover sufficient soil to reduce the pressure to the safe load for the soil will be the total load, including that of the column and slab, divided by the safe load for the soil.

As the dimensions of the slab are not known, its approximate weight can be allowed for, as in the above equation for the column itself; the result can be adjusted if the allowance is found to vary much from the actual amount.

The column and load equals 103 516 lb., allowing 5 000 lb. for the slab, the total load will be 103 516 + 5 000 = 108 516 lb. The safe load for the soil equals 4 000 lb. per square foot, hence the area equals

$$\frac{108\,516}{4\,000} = 27{\cdot}12 \text{ square feet,}$$

say 5 feet 3 inches square.

In deciding upon the thickness of the slab we have to consider the area of concrete required to resist the tendency of the column to shear through; also the amount required to resist the compression from the bending moment of the upward reaction of the soil.

The thickness for the bending moment may be determined first, then if found to be insufficient to take the shear it can be increased, or reinforcement used to take up the excess shear.

To obtain the thickness the greatest bending moment must be determined. This will take effect around the edge of the column base, and will equal the reaction of the soil, under the portion outside the column base, multiplied by the average leverage. Each side of the slab can be considered a cantilever fixed at the edge of the column base, as the stress each side will be equal only one side need be considered. The average reaction of the soil, or the pressure from the slab, is 4 000 lb. per square foot. By referring to Fig. 103 the bending moment along the line A B may be taken as that of the reaction of the soil under the portion A B C D, and will

equal all the area of that portion multiplied by 4 000 lb., and then by the distance of the center of gravity of the figure A B C D from the line A B, this distance is obtained in an easy manner by making A F and B E equal, D C and D G and C H equal A B, the diagonals will intersect at the center of gravity. This distance can also be obtained by the following formula :—

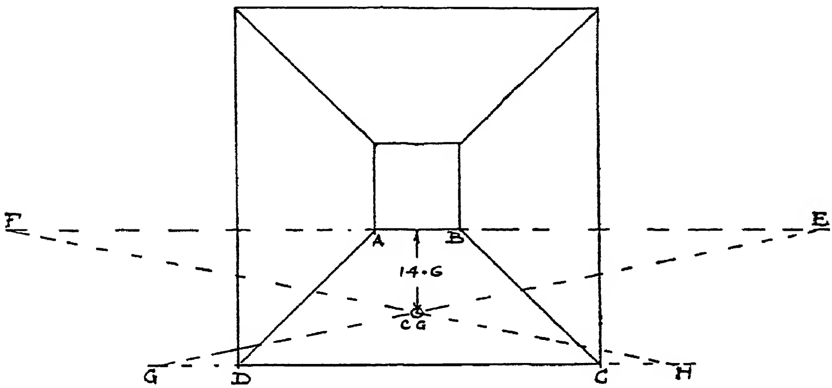


FIG. 103.

Let A B $= a$, C D $= b$, and the projection of the slab $= c$ Then the distance of the center of gravity from the line A B, will equal

$$\frac{c\,(a+2\,b)}{3\,(a+b)},$$

which, for the base of the column at 12 inches, equals

$$\frac{25 \cdot 5\,(12+2\times 63)}{3\,(12+63)} = 15 \cdot 64 \text{ inches.}$$

The reaction of the soil equals

$$\frac{5 \cdot 25+1}{2} \times \frac{25 \cdot 5}{12} \times 4\,000 = 26\,562 \text{ lb.}$$

Then M equals $26\,562 \times 15 \cdot 64 = 415\,130$ inch-lb.

The formulas for beams will now apply; therefore,

$$d=\sqrt{\frac{M}{95\,b}},$$

the width of the slab at the column base equals $b=$ 12 inches, then

$$d=\sqrt{\frac{415\,130}{95\times 12}}=$$

say 19 inches, or a full thickness of 21 inches. $a=pbd=$, $0{\cdot}006\,75\times 12\times 19=1{\cdot}54$ square inches. Say 7 bars ½ inch square spaced across the whole width.

The slab need not be 21 inches thick throughout, it can be reduced to 12 inches, as shown by Fig. 104.

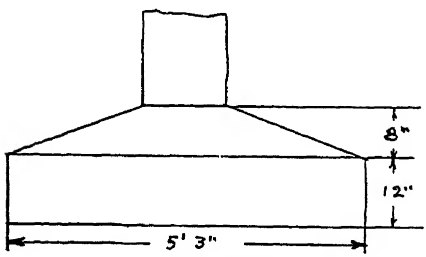


Fig. 104.

The area of concrete resisting the tendency of the column to shear through the slab equals the perimeter of the base of the column multiplied by the thickness of the slab; this should be sufficient to take at least half the shear, the remainder being resisted by the reinforcement. The total shear equals the total reaction of the soil less the portion directly under the column base, it will therefore be, $108\,516 - 4\,000\times 1$, say 104 500 lb. The amount the concrete will take at 60 lb. per square inch equals $4\times 12\times 19\times 60 = 54\,720$ lb. The balance for reinforcement equals $104\,500 - 54\,720 = 49\,780$ lb., which must be provided for by reinforcement, or by increasing the depth of the slab, or by increasing the width of the base of the column to give it a larger perimeter, thus increasing the punching shear area.

The width of the column base to provide sufficient concrete to take all the shear can be determined as follows:—The shear reaction equals 104 500 lb., this

divided by the allowable unit shear multiplied by the thickness of the slab will equal the perimeter of the base, which divided by four will give the diameter. The diameter of the base will therefore equal

$$\frac{104\ 500}{60 \times 19 \times 4},$$

say 23 inches, as shown in Fig. 105. With this increased base the reinforcement in the slab can be reduced, as the lever arm for the bending moment will be shortened.

For the 19 inch slab and the 12 inch column base, the balance of shear around the column base to be taken by reinforcement is already found to be 49 780 lb. Using stirrups inclined at 45 degrees the area of steel required equals $\frac{S}{\sqrt{2}\,t} = \frac{49\ 780}{1{\cdot}414 \times 16\ 000} = 2{\cdot}2$ square inches. Say 3 stirrups each side, out of $\frac{1}{2}$ inch by $\frac{3}{16}$ inch flat steel.

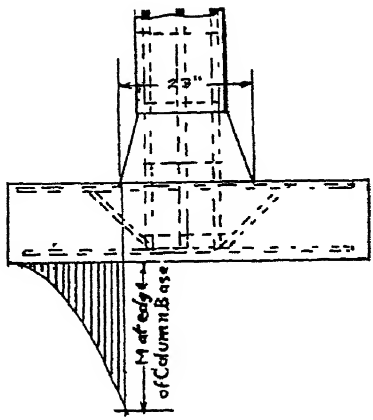


FIG. 105.

If a thinner foundation slab is desirable it can be designed in a similar manner to a doubly reinforced beam, as follows :—Spreading the column base to 18 inches, and limiting the depth to, say 12 inches, we have a case in which the concrete is less than that required for the economic section ; therefore, double reinforcement will be necessary. The tension rods can be designed by the formula

$$a = \frac{M}{t\left(d - \frac{n}{3}\right)}$$

and the compression rods by the formula

$$A_c = \frac{3\,(2\,t\,a - c\,b\,n)}{4\,c\,m}$$

(see page 118). The bending moment will not be the same as for the thicker slab as the projection is less ; using the formula as before, the lever arm

$$= \frac{c\,(a+2\,b)}{3\,(a+b)} = \frac{22{\cdot}5(18\ +2\times 63)}{3\,(18+63)} = 13\tfrac{1}{3} \text{ inches.}$$

The reaction of the soil equals

$$\frac{5{\cdot}25+1{\cdot}5}{2} \times 1{\cdot}88 \times 4\ 000 = 25\ 380 \text{ lb.}$$

Then $M = 25\ 380 \times 13\frac{1}{3} = 338\ 400$ inch-lb. Taking the effective depth as $10\frac{1}{2}$ inches, then for stresses of 600 and 16 000, $n = 0{\cdot}36 \times 10\frac{1}{2} = 3{\cdot}78$ inches ; hence,

$$a = \frac{338\ 400}{16\ 000\ \left(10{\cdot}5 - \dfrac{3{\cdot}78}{3}\right)} = 2{\cdot}28 \text{ square inches.}$$

$$A_c = \frac{3\,(2\times 16\ 000\times 2{\cdot}28 - 600\times 18\times 3{\cdot}78)}{4\times 600\times 15} = 2{\cdot}68 \text{ square}$$

inches, which must not be placed further from the surface than $\frac{1}{3}\,n$ (see page 117). This reinforcement is for the width of 18 inches ; we may therefore say, for the tension reinforcement, $\frac{7}{8}$ inch square rods at 7 inch centers, and for the compressive reinforcement, $\frac{7}{8}$ inch at 8 inch centers.

For the 12 inch slab and the 18 inch base the concrete will take $18\times 4\times 12\times 60 = 51\ 840$ lb. shear, stirrups are required for the remainder. Hence the area of steel required equals

$$\frac{104\ 500 - 51\ 840}{1{\cdot}414\times 16\ 000} = 2{\cdot}33 \text{ square inches.}$$

The same stirrups as for the thick slab will do, arranged as shown in Fig. 105.

Theoretically the bars can be reduced towards the edges of the slab, as the bending moment diminishes as the ordinates of the parabolic diagram, Fig. 105, but to guard against workmen's possible errors in placing the bars is is advisable to keep them all the same size and distance apart, and placed at right angles. Alternate bars, however, need not run more than half-way across from the column base to the outside of the slab, as shown in Fig. 106.

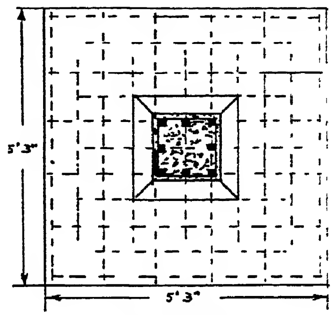


Fig. 106.

HOOPED COLUMNS.

The strength of columns with simple binding has been considered, the binding being spaced at a distance sufficient to prevent buckling of the vertical reinforcement. If the spacing of the binding is not more than 0·6 of the diameter of the column within the binding, it will also prevent the lateral expansion of the concrete, and thus increase its strength. The amount of increase depends upon :—(A) the arrangement of the vertical rods, whether in the form of a rectangle or a circle ; (B) the form of the binding, whether horizontal hoops or spirals ; (C) the ratio of volume of binding to volume of concrete within the binding.

Numerous tests have shown that the compressive strength of columns is increased from 500 to 1 000 lb. per square inch for each percentage of hooping employed. Considere and other authorities have shown the hooping to be equivalent to as much as 2·4 times the same quantity of longitudinal steel.

According to the report of the Royal Institute of British Architect's Joint Committee on Reinforced Concrete, there is no gain in strength if the distance

between the hoops exceeds $0 \cdot 6\ d$, and the distance should not be less than $0 \cdot 2\ d$, where d equals the diameter of the column within the hoops. The increase of strength due to the hooping is given as $c\,f\,s\,r$, where c = the stress for the concrete not hooped ; f = a factor depending on the form of binding ; s = a factor depending upon the distance apart of the hoops or spirals ; r = the ratio of the volume of binding to the volume of concrete within the binding, which equals $\frac{V_h}{V}$, taken for any portion of the length of the column. V_h = volume of hooping reinforcement in cubic inches, V = volume of concrete within the hooping in cubic inches.

The volume of curvilinear hooping should never be less than 0·5 per cent. of the volume of concrete within the hooping. The diameter of rectilinear hooping should be not less than $\frac{3}{16}$ inch.

From the above it follows that the resistance of the concrete per square inch equals c plus the increase, it will therefore be $c + c\,f\,s\,r = c\,(1 + f\,s\,r)$.

The above-mentioned report gives the following values for f and s where p = pitch of the spirals, or the distance apart of the hoops, in terms of the diameter :—

p	s	f
$0 \cdot 2\ d$	32	Spirals = 1·00
$0 \cdot 3\ d$	24	Circular hoops = 0·75
$0 \cdot 4\ d$	16	Rectilinear hoops = 0·50
$0 \cdot 5\ d$	8	
$0 \cdot 6\ d$	0	

Or s can be obtained from the equation

$$s = 48 - 80\,\frac{p}{d}.$$

The same report also gives the following limits of stress to be observed in columns :—

(A) The stress on the metal reinforcement (i.e., m times the stress on the concrete) should not exceed 0·5 of the yield-point of the metal.

(B) Whatever the percentage of the hooping, the working stress on the concrete should not exceed $(0{\cdot}34+0{\cdot}32\,f)$ times the ultimate crushing resistance of the concrete.

For rectilinear hooping	$0{\cdot}34+0{\cdot}32\,f$	=	0·50
,, circular ,,	,,	=	0·58
,, spiral ,,	,,	=	0·66

From the foregoing, it follows that if the concrete has an ultimate resistance of 2 400 lb. per square inch, with efficient rectilinear hooping, the working stress may be raised from 500 lb. to 1 200 lb. per square inch. With circular hooping it may be raised to 1 400 lb. per square inch, and with spiral hooping to 1 600 lb. per square inch, providing 0·5 of the yield-point of the metal is not exceeded.

The working load for a short column not hooped equals $c\,[A+(m-1)\,A_c]$. For a hooped column the working stress equals $c\,(1+f\,s\,r)$; substituting this value for c in the equation for columns not hooped, we get for a hooped column,

$$W = c\,(1+f\,s\,r)\,[A+(m-1)\,A_c].$$

If the length exceeds 15 times the diameter within the hooping, Gordon's formula must be used, as previously explained.

The American Joint Committee on Reinforced Concrete recommend the following :—

Columns reinforced with not less than 1 per cent., and not more than 4 per cent. of longitudinal bars, and with circular hoops or spirals not less than 1 per cent. of the volume of concrete, a unit stress 55 per cent. higher than given for columns reinforced with longitudinal bars only, for which the permissible stress is given as 22½ per cent. of the compressive strength of the concrete, provided the ratio of unsupported length of column to diameter of the hooped core is not more than 10. This report also recommends :—

(a) That the minimum size of columns to which the working stresses may be applied be 12 inches out to out.

(b) In all cases the longitudinal reinforcement is to be assumed to carry its proportion of stress according to the moduli of elasticity. The hoops or bands are not to be counted on directly as adding to the strength of the column.

(c) The total amount of hooping shall not be less than 1 per cent. of the volume of concrete enclosed. The clear spacing of such hooping shall not be greater than one-sixth the diameter of the enclosed column and preferably not greater than one-tenth, and in no case more than $2\frac{1}{2}$ inches. Hooping is to be circular, and the ends of bands must be united in such a way as to develop their full strength.

(d) The strength of hooped columns depends very much upon the ratio of length to diameter of hooped core and the strength due to the hooping rapidly decreases as this ratio increases beyond five. The recommendations are for a length of not more than ten diameters of the hooped core.

The New York City Building Code allows an axial compression of 500 lb. per square inch on the concrete within the hoops or spirals, and 7 500 lb. (m times 500 lb.) per square inch on the vertical reinforcement, plus a load per square inch on the effective area of the concrete equal to twice the percentage of lateral reinforcement multiplied by the permissible tensile stress in the reinforcement, for columns having not less than 0·5 per cent. nor more than 2 per cent. of hoops or spirals spaced not further apart than one-sixth the diameter of the enclosed column nor more than 3 inches, and having not less than 1 per cent. nor more than 4 per cent. of vertical reinforcement.

Cleveland and St. Louis allow 2·4 times the volume of hooping to be considered as longitudinal reinforcement. Cincinnati 2·2 times, and Chicago 2·5 times. The Los Angeles Building Code allows 550 lb for unhooped columns and 800 lb. for hooped columns.

Example XXXIX.—Determine the working load for a short square column reinforced with four 1-inch square bars, bound with horizontal hoops of $\frac{3}{16}$ inch wire placed 4 inches apart ; the diameter inside the hoops being 10 inches.

By the formulas recommended by the R.I.B.A. Joint Committee, the working load equals

$$W = c\ (1 + f\ s\ r)\ [A + (m - 1)\ A_c].$$

Taking $c = 500$, $m = 15$, A = area of concrete inside the hoops, = 100 square inches, A_c = area of longitudinals = 4 square inches, f (for rectilinear hoops) = 0·5, $s = 48 - 80\ \frac{p}{d}$, $p = 4$ inches, and $d = 10$ inches ; hence $\frac{p}{d} = 0{\cdot}4$; therefore $s = 48 - 80 \times 0{\cdot}4 = 16$, as given on page 218 ; r = ratio of volume of hooping to volume of concrete $= \frac{V_h}{V}$. As the hoops are 4 inches apart the volume of concrete to one hoop equals $10 \times 10 \times 4 = 400$, the volume of one hoop equals area multiplied by the length, $= \frac{3}{16} \times \frac{3}{16} \times 10 \times 4 = 1{\cdot}4$:

hence $$r = \frac{1{\cdot}4}{400} = 0{\cdot}0035.$$

Filling in the values we get :—

$$W = 500\ (1 + 0{\cdot}5 \times 16 \times 0{\cdot}0035)\ (100 - 14 \cdot 4) =$$
$$500 \times 1{\cdot}028 \times 156 = 80\ 184 \text{ lb.}$$

If the column is not hooped,

$$W = c\ (A + 14\ A_c) = 500\ (100 + 14 \times 4) = 78\ 000 \text{ lb.}$$

We therefore gain 2 184 lb. by using $\frac{3}{16}$ inch hoops 4 inches apart. There would be a greater gain if we place the hoops 3 inches apart, or use thicker wire.

Example XL.—Compare the working load, determined by the R.I.B.A. recommendations, and the New York Building Code requirements, for a circular column reinforced with eight $\frac{3}{4}$ inch round rods, with spiral binding at 3 inch pitch out of $\frac{1}{4}$ inch square wire; the diameter within the binding equals 15 inches.

BY THE R.I.B.A. RECOMMENDATIONS.

$W = c(1+fsr)[A+(m-1)A_c]$. $c = 500$; $m = 15$; $A = 15\times15\times0{\cdot}785\,4 = 176{\cdot}7$; $A_c = 8\times0{\cdot}75\times0{\cdot}75\times0{\cdot}785\,4 = 3{\cdot}534$; $f = 1$; $s = 48 - 80\frac{p}{d}$, $p = 3$; $d = 15$; hence $s = 48 - 80\times\frac{3}{15} = 32$.

$r = \frac{V_h}{V}$, taking the volume for one revolution of the binding, $V = 15\times15\times0{\cdot}785\,4\times3 = 530$.

V_h = sectional area of the building multiplied by the length of one revolution. The latter will equal the square root of the sum of the squares of the circumference and pitch

$$= \sqrt{(15\times3{\cdot}1416)^2+3^2} = 47{\cdot}21\,;$$

hence, $V_h = 47{\cdot}21\times0{\cdot}25\times0{\cdot}25 = 2{\cdot}95$.

Then $$r = \frac{2{\cdot}95}{530} = 0{\cdot}005\,5.$$

Now filling in the values we get :—

$W = 500\,(1 + 1 \times 32 \times 0{\cdot}005\,5)\,(176{\cdot}7 + 14 \times 3{\cdot}53) = 500 \times 1{\cdot}176 \times 226{\cdot}12 = 132\,958{\cdot}56$ lb.

BY THE NEW YORK BUILDING CODE REQUIREMENTS.

$W = c\,(A+14\,A)+2\,p\,A\,7\,500$; where p equals the percentage of lateral reinforcement $= \frac{2{\cdot}95}{530} = 0{\cdot}005\,57$.

Then $W = 500\,(176{\cdot}7+14\times3{\cdot}534)+2\times0{\cdot}00557\times176{\cdot}7\times7\,500 = 127\,851$ lb., about 5 100 lb. less than by the R.I.B.A. rule.

Without hooping the working load would be:— c (A+14 A_c) = 500 (176·7+14×3·53) = 113 060 lb. By hooping with $\frac{1}{4}$ inch wire at 3 inch pitch, we therefore gain about 19 900 lb. by one rule and 14 800 lb. by the other.

Example XLI.—Determine the necessary reinforcement, by the same rules as for the last example, for a circular column 12 inches diameter, within the binding, to carry a load of 120 000 lb.; the binding to consist of hoops of $\frac{1}{4}$ inch square wire, placed 3 inches apart.

BY THE R.I.B.A. FORMULAS.

$$W = c\,(1+f\,s\,r)\,[A+(m-1)\,A_c],$$

we get
$$A = \frac{W - A\,c\,(1+f\,s\,r)}{c\,(m-1)\,(1+f\,s\,r)},$$

for which W = 120 000 lb.; A = 12×12×0·7854 = 113; c = 500; m = 15; f = 0·75 (see page 218);

$$s = 48 - 80\frac{p}{d} = 48 - 80 \times \tfrac{3}{12} = 28;$$

$r = \frac{V_h}{V}$, taking the volume for one hoop, then r equals

$$\frac{12 \times 3{\cdot}1416 \times 0{\cdot}25 \times 0{\cdot}25}{12 \times 12 \times 0{\cdot}7854 \times 3} = 0{\cdot}007.$$

Hence

$$A = \frac{120\,000 - 113 \times 500\,(1+0{\cdot}75 \times 28 \times 0{\cdot}007)}{14 \times 500\,(1+0{\cdot}75 \times 28 \times 0{\cdot}007)}$$

$$= \frac{55\,195}{8\,029} = 6{\cdot}87 \text{ square inches.}$$

Say 8 round rods $1\frac{1}{16}$ inch diameter.

BY THE RULES OF THE NEW YORK BUILDING CODE.

$W = c\,(A + 14\,A_c) + 2\,p\,A\;7\,500$, from which we get :—

$$A = \frac{W - (c\,A + 2\,p\,A\;7\,500)}{14\,c}$$

$$= \frac{120\,000 - (500 \times 113 + 2 \times 0{\cdot}007 \times 113 \times 7\,500)}{14 \times 500}$$

$$= 7{\cdot}37 \text{ square inches.}$$

Or 0·5 square inch more than by the R.I.B.A. rule.

Example XLII.—Design the section of a hooped circular column to support a load of 200 000 lb., by both the R.I.B.A. rules and the New York Building Code.

In this case we have to determine the area of concrete, the vertical reinforcement, and the hooping.

BY THE R.I.B.A. RULES.

Area of the vertical rods may be from 0·5 to 10 per cent. of the area of concrete. The volume of hooping must not be less than 0·5 per cent. of the volume of concrete. We may therefore decide on these percentages then determine the section by the formula :—

$$W = A\,c\,[1 + (m - 1)\,p]\,;$$

which, for hooped columns will be

$$W = A\,c\,[1 + (m - 1)]\,p\;\;(1 + f\,s\,r)\,;$$

Hence, $$A = \frac{W}{c\,[1 + (m - 1)\,p]\;\;(1 + f\,s\,r)},$$

$W = 200\,000$ lb. ; $c = 500$; $m = 15$; $s = 48 - 80\frac{p}{d}$, allowing for hoops at $0{\cdot}3\,d$ centers, then $s = 48 - 80 \times 0{\cdot}3 = 24$; f for circular hoops, $= 0{\cdot}75$; $r = \frac{V_h}{V}$, taking the volume of hooping as 0·6 per cent. of the volume of concrete,

then $$r = \frac{0{\cdot}006\,V}{V} = 0{\cdot}006.$$

Let the area of the vertical rods equal 5 per cent. of the

Without hooping the working load would be:— $c\ (A+14\ A_c) = 500\ (176{\cdot}7+14\times 3{\cdot}53) = 113\ 060$ lb. By hooping with $\frac{1}{4}$ inch wire at 3 inch pitch, we therefore gain about 19 900 lb. by one rule and 14 800 lb. by the other.

Example XLI.—Determine the necessary reinforcement, by the same rules as for the last example, for a circular column 12 inches diameter, within the binding, to carry a load of 120 000 lb.; the binding to consist of hoops of $\frac{1}{4}$ inch square wire, placed 3 inches apart.

BY THE R.I.B.A. FORMULAS.

$$W = c\ (1+f\ s\ r)\ [A+(m-1)\ A_c],$$

we get
$$A = \frac{W - A\ c\ (1+f\ s\ r)}{c\ (m-1)\ (1+f\ s\ r)},$$

for which $W = 120\ 000$ lb.; $A = 12\times 12\times 0{\cdot}7854 = 113$; $c = 500$; $m = 15$; $f = 0{\cdot}75$ (see page 218);

$$s = 48 - 80\ \frac{p}{d} = 48 - 80\ \times \tfrac{3}{12} = 28\ ;$$

$r = \dfrac{V_h}{V}$, taking the volume for one hoop, then r equals

$$\frac{12\times 3{\cdot}1416\times 0{\cdot}25\times 0{\cdot}25}{12\times 12\times 0{\cdot}7854\times 3} = 0{\cdot}007.$$

Hence

$$A = \frac{120\ 000 - 113\times 500\ (1+0{\cdot}75\times 28\times 0{\cdot}007)}{14\times 500\ (1+0{\cdot}75\times 28\times 0{\cdot}007)}$$

$$= \frac{55\ 195}{8\ 029} = 6{\cdot}87 \text{ square inches.}$$

Say 8 round rods $1\frac{1}{16}$ inch diameter.

BY THE NEW YORK BUILDING CODE.

$W = c(A + 14 A_c) + 2 \times 7\,500\,pA$, With $A_c = 0{\cdot}05\,A$, and $p = 0{\cdot}006$, we get :—

$W = c(A + 14 \times 0{\cdot}05A) + 2 \times 0{\cdot}006A \times 7\,500$, or

$A\,(c + 14 \times 0{\cdot}05\;c + 2 \times 0{\cdot}006 \times 7\,500)$. From which

$$A = \frac{W}{c + 14 \times 0{\cdot}05\,c + 2 \times 0{\cdot}006 \times 7\,500} =$$

$$\frac{200\,000}{500 + 14 \times 0{\cdot}05 \times 500 + 2 \times 0{\cdot}006 \times 7\,500} = 212.$$

The same as by the R.I.B.A. rules.

ECCENTRICALLY LOADED COLUMNS.

When a column is eccentrically loaded, that is, when the load is placed so that the center of pressure is not over the center of the column, the stress will not be uniformly distributed throughout the section, but will be greatest at the edge nearest the center of pressure of the load.

A column supporting the end of a single girder, although the end may extend across the whole width of the column, the deflection, however slight, would cause the load to become eccentric ; a column supporting two girders which are not in the same straight line or over the center of the column, or with the girders on opposite sides unequally loaded ; a column having a projection, or a bracket fixed to the top, upon which the load rests ; columns not fixed with their axis perfectly vertical, and columns subject to wind pressure or other side thrust, should all be treated as eccentrically loaded. In some cases the exact eccentricity is difficult, and sometimes quite impossible, to determine, as it will be affected by the deflection of the column and girder, or forces not accurately determinable ; for such cases there is no formula that can be applied to determine the eccentricity with sufficient accuracy to be of practical use, the designer must use his own discretion in the matter.

With concrete columns and girders, however, the deflection is so slight that it is usual to ignore it, and determine the stress from the load and the loads eccentricity only. Should these be so great as to cause one side of the column to be in tension, the reinforcement should be designed by an application of the formulas for beams. This is fully explained further on.

For simple cases the maximum and minimum stresses are best determined by equating the bending moment with the modulus of section, thus :—

$$(1)\ \text{Max. } c = \frac{W}{A'} + \frac{W\,y}{Z} \qquad (2)\ \text{Min. } c = \frac{W}{A'} - \frac{W\,y}{Z}.$$

From the above formulas we get :—

$$(3) \qquad W = \frac{\text{Max. } c\,A'\,Z}{Z + A'\,y}$$

A' = the equivalent concrete section, which equals the sectional area of the column plus 14 times the sectional area of the steel. y = the eccentricity. Z = the modulus of section, which is the moment of inertia, I, divided by the distance of the neutral axis from the extreme edge of the section ; A = the equivalent concrete section, it equals $b\,d + (m - 1)\,A_c$.

If Min. c turns out a minus quantity, or if the eccentricity exceeds $\frac{Z}{A'}$, there will be tension in the side furthest from the load, which will be the case when the stress from the bending moment exceeds the stress from the direct thrust, that is, when $\frac{W\,y}{Z}$ exceeds $\frac{W}{A'}$. When these are equal there will be no stress on the edge of the section furthest from the load, and at the nearest edge the stress will be double that from the direct thrust, that is to say, it will equal $2\,\frac{W}{A'}$. When $\frac{W\,y}{Z}$ is less than $\frac{W}{A'}$, there will be compression throughout the whole section.

For a rectangular section not reinforced : the moment of inertia $= \frac{b\,d^3}{12}$,

therefore $$Z = \frac{\frac{b\,d^3}{12}}{\frac{d}{2}} = \frac{b\,d^2}{6}, \text{ and } A' = b\,d,$$

therefore $$\frac{Z}{A'} = \tfrac{1}{6}\,d\,;$$

consequently, to avoid tension in a plain square column, not reinforced, the eccentricity must not exceed $\frac{1}{6}\,d$, or we may say that the center of pressure of the load must not be nearer the edge of the section than $\frac{1}{3}\,d$, or the far side will be in tension.

With a reinforced rectangular section : I equals the moment of inertia of the concrete plus the moment of inertia of the steel, it therefore equals

$$\frac{b\,d^3}{12} + (m - 1)\,A_c\left(\frac{d_1}{2}\right)^2.$$

Hence, $$Z = \frac{\frac{b\,d^3}{12} + (m - 1)\,A_c\left(\frac{d_1}{2}\right)^2}{\frac{d}{2}};$$

further simplified, and taking $m = 15$,

$$Z = \frac{b\,d^3 + 42\,A_c\,d_1^2}{6d}.$$

Where d_1 is the diameter from center to center of the rods, and b and d the effective diameters of the column.

For a circular column with the bars arranged to form a rectangle ; $$Z = \frac{d^4 + 56\,A_c\,d_1^2}{8\,d}.$$

For a circular column with the bars arranged in a circle;

$$Z = \frac{d^4 + 28\ A_c\ d_1^2}{8\ d},$$

where d_1 is the diameter of the circle, which passes through the center of the bars. $\frac{Z}{A'}$ may be a trifle more or less than $\frac{1}{6}\ d$; the variation will depend on the percentage of reinforcement, and the diameter between the rods relative to the diameter of the column, as shown by the following cases :—

(1) Taking a square column 12 inches effective diameter, reinforced with 4 rods, each 1 inch square in section, placed at 10 inch centers.

Being rectangular,

$$Z = \frac{b\ d^3 + 42\ A_c\ d_1^2}{6\ d} = \frac{12^4 + 42 \times 4 \times 10 \times 10}{6 \times 12} = 521 \cdot 3.$$

$$A' = b\ d + (m - 1)\ A_c = 144 + 14 \times 4 = 200$$

Hence $$\frac{Z}{A'} = \frac{521 \cdot 3}{200} = 2 \cdot 6065 \text{ inches.}$$

(2) Taking the same column as No. 1, but placing the rods at 8-inch centers, we get :—

$$Z = \frac{12^4 + 42 \times 4 \times 8^2}{6 \times 12} = 437 \cdot 3.$$

A' will be the same as for No. 1.

Hence $$\frac{Z}{A'} = \frac{437 \cdot 3}{200} = 2 \cdot 1865 \text{ inches.}$$

In both cases $\frac{Z}{'A}$ is greater than $\frac{1}{6}\ d$, but if we place the rods closer together $\frac{A'}{Z}$ will become less than $\frac{1}{6}\ d$.

In (1) there would be tension if the eccentricity exceeded 2·6065 inches. In (2) there would be tension if the eccentricity exceeded 2·18 inches.

Example XLIII.—Determine the maximum and minimum stress in the square column as Fig. 108. It is reinforced with 8 round rods of one-inch diameter, and supports a total load of 90 000 lb., the center of pressure being 3 inches from the axis of the column.

$$\text{Max. } c = \frac{W}{A'} + \frac{W\,y}{Z}. \qquad W = 90\ 000 \text{ lb.}$$

A′ = area of column plus 14 times the area of steel
$= 15 \times 15 + 8 \times 0{\cdot}7854 \times 14 = 313.$

$$Z = \frac{b\,d^3 + 42\,A_c\,d_1^2}{6\,d}.$$

Taking 15 inches as the effective diameter, then

$$Z = \frac{15 \times 15^3 + 42 \times 6{\cdot}28 \times 14^2}{6 \times 15} = 1\ 137.$$

$$\text{Then Max. } c = \frac{90\ 000}{313} + \frac{90\ 000 \times 3}{1\ 137} = 287 + 237 = 524 \text{ lb.}$$

$$\text{Min. } c = \frac{W}{A'} - \frac{W\,y}{Z} = 287 - 237 = 50 \text{ lb.}$$

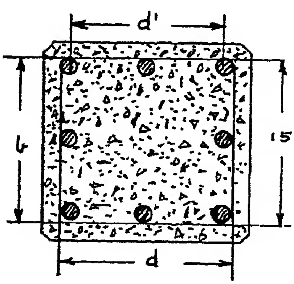


FIG. 108.

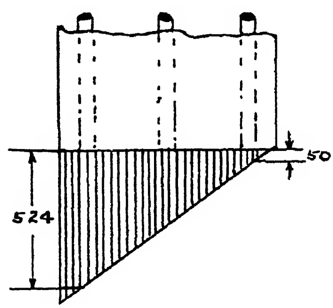


FIG. 109.

With these values a diagram as Fig. 109 can be drawn, from which the stress per square inch at any point across the section can be directly scaled. The stress in any bar will equal *m* times the value of the ordinate under the bar.

Example XLIV.—Determine the load for an 18-inch circular column reinforced with 8 round rods $1\frac{1}{4}$ inch diameter, the diameter from center to center of the rods being 14 inches. The center of pressure of the load is 2 inches from the center of the column. The stress on the concrete is not to exceed 500 lb. per square inch. The effective diameter can be taken as 16 inches.

From page 227 $$W = \frac{\text{Max. } c\, A'\, Z}{Z + A'\, y}.$$

Area of steel = 9·8 inches, d can be taken as 16 inches,

$$A' = 16^2 \times 0{\cdot}7854 + 14 \times 9{\cdot}8 = 338.$$

Then

$$Z = \frac{d^4 + 28\, A_c\, d_1^2}{8\, d} = \frac{16^4 + 28 \times 9{\cdot}8 \times 14 \times 14}{8 \times 16} = \frac{119\,318}{128}$$
$$= 932.$$

For a maximum compression of 500 lb. per square inch,

$$W = \frac{500 \times 338 \times 932}{932 + 338 \times 2} = 97\,953 \text{ lb.}$$

$$\text{Min. } c = \frac{W}{A'} - \frac{W\, y}{Z} = \frac{97\,953}{338} - \frac{97\,953 \cdot 2}{932} = 80 \text{ lb.}$$

Example XLV.—Taking case No. 1 of page 229, and applying a load of 40 000 lb. at the point given in that example, we then have a column 12 inches square reinforced with four rods 1 inch square at 10 inch centers, with an eccentric load of 40 000 lb., the eccentricity being 2·6065 inches.

By the formula for eccentric loads the stresses in the opposite sides equals

$$\frac{A'}{W} \pm \frac{W\, y}{Z}\,; \quad W = 40\,000\,;\, y = 2{\cdot}6065.$$

A′ and Z are already found, in (1), to be 200 and 521·3

respectively. Then the stress in the side nearest the load equals

$$\frac{40\ 000}{200}+\frac{40\ 000\times 2\cdot 6\ 065}{521\cdot 3}=200+200$$

$$=400 \text{ lb. compression per square inch.}$$

The stress in the opposite side equals $200-200=0$.

The stress in the concrete will therefore be as shown by the shaded diagram of Fig. 110.

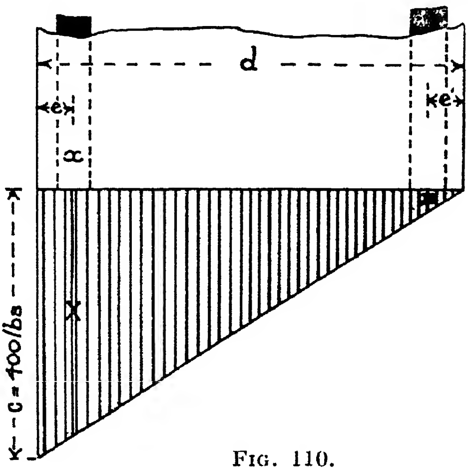


FIG. 110.

It is seldom, if ever, necessary to calculate the stress in the rods, as it cannot exceed m times the stress in the concrete at the same point; consequently, if the concrete is not overstressed, the stress in the steel will be only about half its limit. However, if the stress here is required it can be determined by scaling the ordinate of the stress diagram under the rods and multiplying by m; or it can be obtained by calculation. Referring to Fig. 110, the stress in the rods x equals 15 times the ordinate X.

By calculation, the stress per square inch in the rods x equals

$$c' = m\,c\,\frac{d-c}{d},$$

where c = maximum compression in the concrete, d =

full diameter of the column, $e =$ distance of the rods from the face of the column.

Hence $$c' = 15 \times 400 \frac{12 - 1}{12} = 5\,500 \text{ lb.}$$

The stress in the rods on the other side $= m$ times the ordinate X′

$$= m\,c\,\frac{e'}{d} = 15 \times 400 \times \frac{1}{12}$$
$= 500$ lb. compression per square inch,

practically a negligible quantity for steel.

Example XLVI.—Taking the same case as Example XLV. but applying the load 4 inches from the axis.

As the eccentricity will now be greater than $\frac{Z}{A'}$, there will be tension in the far side.

As in Example XLV.,

$$c = \frac{W}{A'} + \frac{W\,y}{Z}.$$

$W = 40\,000$; $A' = 200$; $Z = 521{\cdot}3$; $y = 4$.

Hence

$$c = \frac{40\,000}{200} + \frac{40\,000 \times 4}{521{\cdot}3} = 200 + 307 = 507 \text{ lb. compression.}$$

The stress in the far side

$$= \frac{W}{A'} - \frac{W\,y}{Z} = 200 - 307 = -107 \text{ lb., tension.}$$

The stress in the concrete throughout the section will be as the shaded diagram Fig. 111.

The compression per inch in the rods on the side nearest the load equals

$$c' = m\,c\,\frac{b - e}{b}.$$

The tension per square inch in the rods on the far side equals

$$t = t' \, m \frac{a - e}{a},$$

a and b are scaled off the diagram. By calculation, when a and b are unknown,

$$c' = \frac{m}{d}\Big[c\,d - e\,(t' + c)\Big] = \frac{15}{12}\Big[507 \times 12 - 1\,(107 + 507)\Big]$$
$$= 6837 \text{ lb. per square inch.}$$

$$t = \frac{m}{d}\Big[t'\,(d - e') - e'\,c\Big] = \frac{15}{12}\Big[107\,(12 - 1) - 1 \times 507\Big]$$
$$= 837 \text{ lb. per square inch.}$$

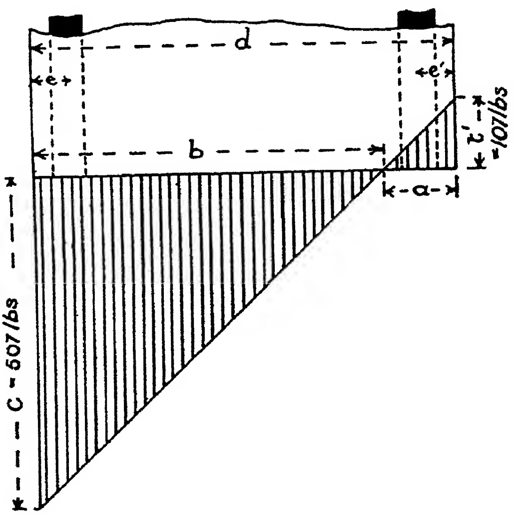


FIG. 111.

The stress in the steel can also be determined by the same formula as that used for the stress in the concrete if we alter the value Z and multiply by m. Then the stress in the rods equals

$$m\left[\frac{W}{A'} \pm \frac{W\,y}{Z'}\right].$$

We obtain Z' by substituting d_1 for d in the denominator of the formula for Z, thus :—

$$Z' = \frac{b\,d^3 + 42\,A_c\,d_1^2}{6\,d_1}.$$

For the case of Example XLVI.,

$$Z' = \frac{12^4 + 42 \times 4 \times 10 \times 10}{6 \times 10} = 625\cdot6.$$

Then

$$c' = m\left[\frac{W}{A'} + \frac{W\,y}{Z'}\right] = 15\left[\frac{40\,000}{200} + \frac{40\,000 \times 4}{625\cdot6}\right]$$
$$= 15\,(200 + 255\cdot8) = 6\,837 \text{ lb.}$$

$$t = m\left[\frac{W}{A'} - \frac{W\,y}{Z'}\right] = 15\,(200 - 255\cdot8) = -837 \text{ lb.} = \text{tension.}$$

Both values are the same as previously found.

Although it is usual to have the same quantity of reinforcement in the far side as in the near side of an eccentrically loaded column, we see, from the foregoing examples, that it is not economical for it to be so, as the stress here will always be less than in the near side, and when the eccentricity equals $\frac{Z}{A'}$, there will be practically no stress in the steel on the far side.

The compression and tension determined by the formulas in Example XLVI. will be correct if the concrete is capable of resisting the tension, and there is no doubt it will do so within a certain limit. In beams this limit is far exceeded, we therefore assume that the concrete has no tensional resistance ; but in ordinary cases of columns if there is tension it will seldom exceed the safe limit for the concrete, which we may take as 200 lb. per square inch. If the eccentricity is sufficient to cause a greater tensional stress than 200 lb. per square inch, the column should be designed in a similar manner to a beam. To do this we must first determine the value for c and t, for we have a direct thrust and a bending

moment to consider; the direct thrust will cause compression throughout the section, which must be allowed for in fixing the values for c and t in the beam formulas. The compression per square inch in the concrete from the direct thrust will be

$$\frac{W}{A+(m-1)A_c},$$

and the compression per square inch in the steel will be

$$m\frac{W}{A+(m-1)A_c}.$$

But usually when we require to fix these values the area of steel is unknown; it is therefore usual to consider the direct thrust as $\frac{W}{A}$ per square inch for the concrete, and m times $\frac{W}{A}$ for the steel. When this is done it will simplify the calculations, and will err slightly on the safe side, as it is allowing for a trifle more than really exists.

Then in the beam formulas c must equal $600-\frac{W}{A}$, and t must equal $16\,000+m\frac{W}{A}$. (Plus because the direct thrust is compression, which must be overcome before there will be tension.)

In most cases we can obtain a more accurate result when we determine the direct thrust, if we assume a reinforcement, as done in the following example:—

Example XLVII.—Determine the reinforcement required for a column 19 inches square to support a beam which transmits a load of 56 000 lb. to the column, the center of pressure being 8 inches from the axis of the column, as Fig. 112.

The compression due to the direct thrust equals $\frac{W}{A'}$.

Allowing for 4 square inches of steel, and an effective diameter of 18 inches, we get :—

Direct thrust equals $\frac{56\ 000}{18^2+14\times 4} = 147$ lb. per square inch.

Taking the safe compression for the concrete as 600 lb.

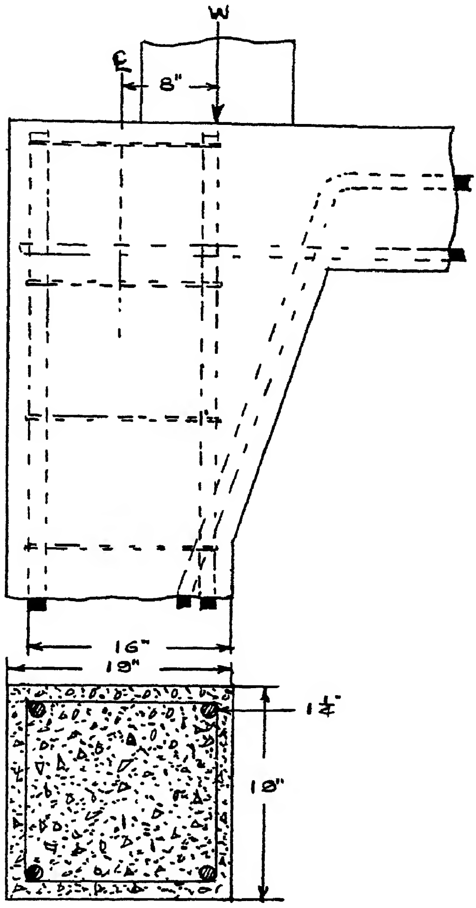


Fig. 112.

per square inch, we have a balance for the concrete to resist the bending moment of 600 — 147 = 453 lb., which will be the value for c. The value for t will be the safe tension for the steel plus m times the compression due to the direct thrust. Therefore $t = 16\ 000 + 15 \times 147 = 18\ 205$ lb.

As the load acts 8 inches from the axis we have a bending moment of 8 W = 8 × 56 000 = 448 000 lb.

We may now design the reinforcement as if it were for a beam from the following data :—

$$M = 448\,000 \text{ lb.} \quad c = 453 \text{ lb.} \quad t = 18\,205 \text{ lb.}$$

We have here a case where the section of the concrete is fixed and the stresses limited, with a given bending moment.

If we place the compression bars at $\frac{n}{3}$ from the compression edge, we can determine the area of steel required for both the compression and tension by equations 1 to 4, pages 117, 118. If we place the compression bars in any other position, we must use the formulas for designing beams with double reinforcement as given in Example XIII. The former method is the easier, and in most cases $\frac{n}{3}$ works out at a suitable depth for the compression bars. If designed for this position they must not be placed nearer to the axis ; if further away, the beam will be a trifle stronger. But by either of these methods the compression bars might not work out to equal the tension bars, while in columns it is usual to make them so, although it is not necessary. If they are required to be equal, the most simple way to determine their area is to take a trial section, and work out the stresses to determine its sufficiency or otherwise. Thus, trying $a = A_c = 2$ inches, then by the formulas for beams with double reinforcement,

$$n = \sqrt{\frac{2m(ad + A_c y)}{b} + \left[\frac{m(a+A_c)}{b}\right]^2} - \frac{m(a+A_c)}{b} =$$

$$\sqrt{\frac{2 \times 15 (2 \times 16 + 2 \times 2)}{18} + \left(\frac{15 \times 4}{18}\right)^2} - \frac{15 \times 4}{18} = 5 \cdot 1 \text{ inches.}$$

$$c = \frac{2\,M}{bn\left(d - \frac{n}{3}\right) + 2\,m\,A_c\,\frac{n-y}{n}(d-y)} =$$

$$\frac{2 \times 448\ 000}{18 \times 5{\cdot}1 \left(16 - \frac{5{\cdot}1}{3}\right) + 2 \times 15 \times 2\, \frac{5{\cdot}1 - 2}{5{\cdot}1} (16 - 2)} = 493 \text{lb.}$$

$$t = \frac{c\, m\, (d - n)}{n} = \frac{493 \times 15\, (16 - 5{\cdot}1)}{5{\cdot}1} = 15\,805 \text{ lb.}$$

The concrete is therefore a trifle overstressed, as c should not exceed 453 lb. By trying $1\frac{1}{4}$ inch round bars the stress in the concrete works out at 440 lb. This size must therefore be used. The elevation and section will be as Fig. 112.

Economic Section.—If b or d were not fixed, the economic section for equal reinforcement could be determined by the formulas :—

$$\frac{\mathrm{M}}{c\, b\, d^2} = \frac{t \frac{n}{d}\left(3 - \frac{n}{d}\right)}{6\, t - 4\, m\, c},\ a = p\, b\, d, \text{ or } a = \frac{3\, c\, b\, n}{6\, t - 4\, m\, c}$$

where
$$\frac{n}{d} = \frac{m\, c}{t + m\, c}.$$

For this case, taking $c = 453$, and $t = 18\,205$,

$$\frac{n}{d} = \frac{15 \times 453}{18\,205 + 15 \times 453} = 0{\cdot}2\,718.$$

$$\text{Then } \frac{\mathrm{M}}{c\, b\, d^2} = \frac{18\,205 \times 0{\cdot}2\,718\, (3 - 0{\cdot}2\,718)}{6 \times 18\,205 - 4 \times 15 \times 453} = 0{\cdot}1\,645.$$

Assuming b to be fixed, but not d, then

$$d = \sqrt{\frac{\mathrm{M}}{0{\cdot}1\,645\, c\, b}} = \sqrt{\frac{448\,000}{0{\cdot}1\,645 \times 453 \times 18}} = 18\tfrac{1}{4} \text{ inches.}$$

$$a = \frac{3\, c\, b\, n}{6\, t - 4\, m\, c},\ \frac{n}{d} = 0{\cdot}2\,718\,;$$

therefore $n = 0{\cdot}2\,718\, d = 4{\cdot}96$.

Hence

$$a = \frac{3 \times 453 \times 18 \times 4 \cdot 96}{6 \times 18\,205 - 4 \times 15 \times 453} = 1 \cdot 478 \text{ square inches,}$$

and $A_t = a$. The total area of steel required, therefore, equals $2 \times 1 \cdot 478$, say 3 inches.

The compression bars must not be placed further from the compression edge than $\frac{n}{3} = 1 \cdot 63$ inches.

The economic section, therefore, requires an effective depth of $18\frac{1}{4}$ inches, which by placing the bars $1\frac{1}{2}$ inches from the edge, will give the column a full depth of 20 inches, with a breadth of 18 inches.

By increasing the depth 2 inches, we would therefore require 3 square inches of reinforcement instead of 4·9 square inches.

Example XLVIII.—Determine the reinforcement required for a column 15 inches square to support a load of 25 000 lb., the center of pressure being 6 inches from the axis of the column.

This is a case similar to Example XLVII., where the section is fixed with the load acting outside the middle third ; therefore we must first determine the maximum value for c and t.

Neglecting the direct thrust taken by the bars, which in this case will be small, we get :—Compression due to the direct thrust $\frac{25\,000}{15 \times 15}$, say 110 per square inch. The balance to resist bending $= c = 600 - 110 = 490$ lb. $t = 16\,000 + 15 \times 110 = 17\,650$ lb.

We now have to determine the reinforcement, the compression bars to equal the tension bars.

In cases of this description it will simplify matters if we know whether the section is more or less than required to take the load when the economic percentage of steel is added. If it is less, we must determine the correct reinforcement as explained for example XLVII. If the

concrete is more than required, we can determine the reinforcement by the equation,

$$a = \frac{M}{t\left(d - \frac{n}{3}\right)}.$$

We can determine which formula to use by comparing the bending moment with the bending moment that the economic section would take. The latter equals

$$\frac{c\,b\,d^2\,t\,\frac{n}{d}\left(3 - \frac{n}{d}\right)}{6\,t - 4\,m\,c},$$

for which

$$n = \frac{m\,c\,d}{t + m\,c}, \text{ or } \frac{n}{d} = \frac{m\,c}{t + m\,c}.$$

Then $$\frac{n}{d} = \frac{15 \times 490}{17\,650 + 15 \times 490} = 0{\cdot}294.$$

Allowing $1\frac{1}{2}$ inch cover to the tension rods, $d = 13{\cdot}5$ inches. Then the bending moment for the economic section equals,

$$\frac{490 \times 15 \times 13{\cdot}5^2 \times 17\,650 \times 0{\cdot}294\,(3 - 0{\cdot}294)}{6 \times 17\,650 - 4 \times 15 \times 490} = 245\,875$$

inch-lb.

The bending moment of the column equals the load multiplied by the eccentricity $= 25\,000 \times 6 = 150\,000$ inch-lb. As this is much less than what the economic reinforcement will develop, we can determine the actual reinforcement by the equation

$$a = \frac{M}{t\left(d - \frac{n}{3}\right)}.$$

$\frac{n}{d} = 0{\cdot}294$; therefore $n = 0{\cdot}294\,d = 3{\cdot}97$ inches.

Then $a = \dfrac{150\ 000}{17\ 650\left(13{\cdot}5 - \dfrac{3{\cdot}97}{3}\right)} = 0{\cdot}7$ square inches,

$A_c = a = 0{\cdot}7$ square inch.

It will be noticed that the latter formula is the same as used to determine a for beams with single reinforcement, where the area of concrete is more than required to form the economic section, and gives the same value for a, whether for single or double reinforcement. From this it is obvious that compressive reinforcement is not really necessary, for without it the concrete will not be overstressed, and if it is used the stress in the concrete will be still further reduced. This statement can be proved by taking the present case and working out the stresses by the formulas for beams with double reinforcement, taking $A_c = a = 0{\cdot}7$, and comparing the results with those obtained by the formulas for beams with single reinforcement, taking $a = 0{\cdot}7$. Thus, for double reinforcement :—

$$n = \sqrt{\frac{2\,m\,(a\,d + A_c\,y)}{b} + \left[\frac{m\,(a + A_c)}{b}\right]^2} - \frac{m\,(a + A_c)}{b}.$$

Taking $y = 1{\cdot}5$, then

$$n = \sqrt{\frac{2 \times 15(0{\cdot}7 \times 13{\cdot}5 + 0{\cdot}7 \times 1{\cdot}5)}{15} + \left(\frac{15 \times 1{\cdot}4}{15}\right)^2} - \frac{15 \times 1{\cdot}4}{15}$$
$$= 3{\cdot}39.$$

$$c = \frac{2\,M}{b\,n\left(d - \dfrac{n}{3}\right) + 2\,m\,A_c\,\dfrac{n-y}{n}\,(d-y)} =$$

$$\frac{2 \times 150\ 000}{15 \times 3{\cdot}39\left(13{\cdot}5 - \dfrac{3{\cdot}39}{3}\right) + 2 \times 15 \times 0{\cdot}7\dfrac{3{\cdot}39 - 1{\cdot}5}{3{\cdot}39}(13{\cdot}5 - 1{\cdot}5)}$$
$$= 390\ \text{lb}.$$

PILES.

Reinforced concrete piles are more expensive than timber piles, but they have the advantage of being practically indestructible, if made of first class material and workmanship (see article on sea-water and alkali), and they can be made to support about twice as much as timber piles ; their durability and economy of maintenance also render them far superior to steel piles.

The methods adopted for the design of reinforced piles are practically the same as those described for columns. There are several methods of building and reinforcing the piles, some are built above ground and others in place. Those built above ground may be straight or tapered and square or circular in cross section, with reinforcement consisting of a number of vertical rods arranged symmetrically around the axis of the pile, the whole being bound with horizontal or spiral reinforcement, as in a column.

If the length of the pile exceeds 20 times its least diameter the reinforcement should be increased at the center to provide for stresses due to handling, and to possible bending, by the pile acting as a long column. In the case of friction piles, i.e., those that will depend for their support upon the friction of the surrounding soil and not from solid material upon which they rest, additional reinforcement against bending after being driven is not necessary. These piles are concreted in horizontal forms, and have a shoe attached. The method of driving is practically the same as for timber piles, except that a cushion of wood, rope, or other material is placed on the head to cushion the blow.

The Raymond System.—The Raymond system of concrete piling consists of a collapsible steel core, tapering from 8 inches at the bottom at the rate of 0·4 inch per foot of length, a pile 20 feet long being 16 inches in diameter at the top. The limit of length for a standard pile is 37 feet 6 inches. Over the core is placed a spirally reinforced sheet-metal shell, the reinforcement of which is grooved into the sheet metal on 3-inch centers the

whole length of the core. The combined core and shell is driven into the soil to a satisfactory refusal ; the core is then collapsed and withdrawn from the shell which is then inspected for bulging or other defects, if found perfect, the reinforcement is placed in position and the concrete poured. The taper of the shell combined with the friction between the shell and surrounding soil considerably increases the carrying power of the pile. The usual load for these piles varies from 25 to 30 tons.

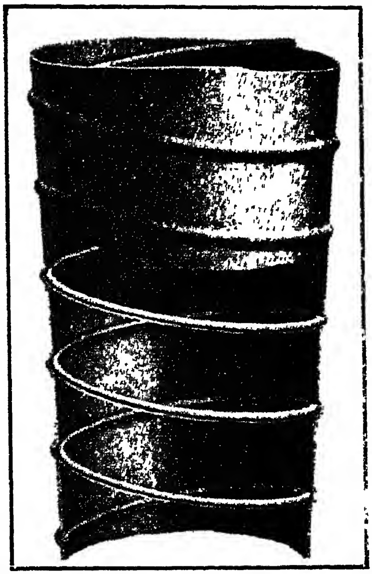


FIG. 114.
Showing the construction of the Spiral Shell of a Raymond Concrete Pile.

Where Raymond piles are used for vertical loads only they are usually not reinforced, but if the surrounding soil is of such a nature that it is likely to slip, or if there is danger of side thrust, hydro-static head or cantilever action on the pile, they are reinforced.

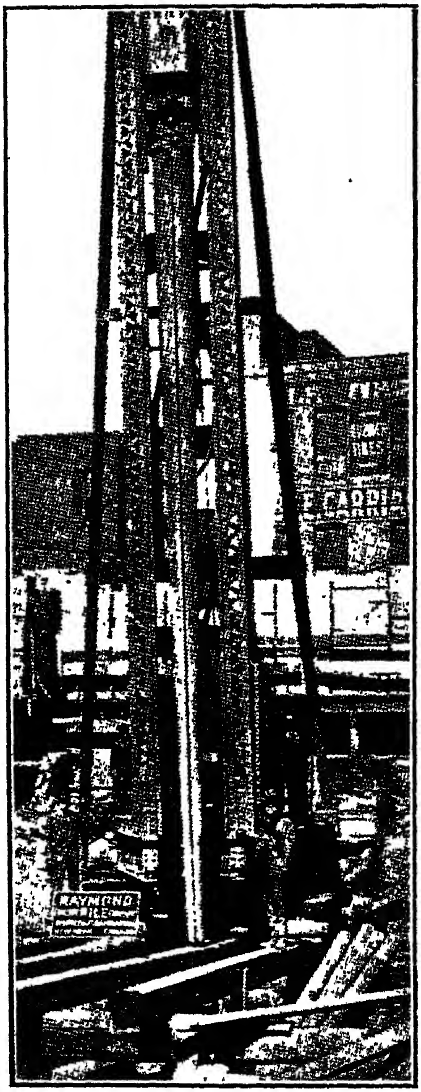


FIG. 115.
A view of the Collapsible Steel Core of a Raymond Concrete Pile, ready to be encased in the Spiral Shell.

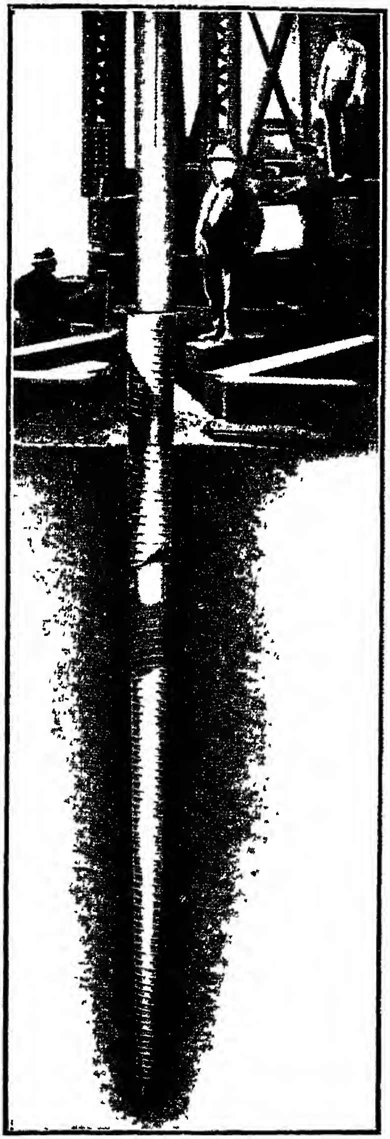


FIG. 116.

Showing the Core of a Raymond Concrete Pile being withdrawn from the Shell after being driven to the desired point of resistance.

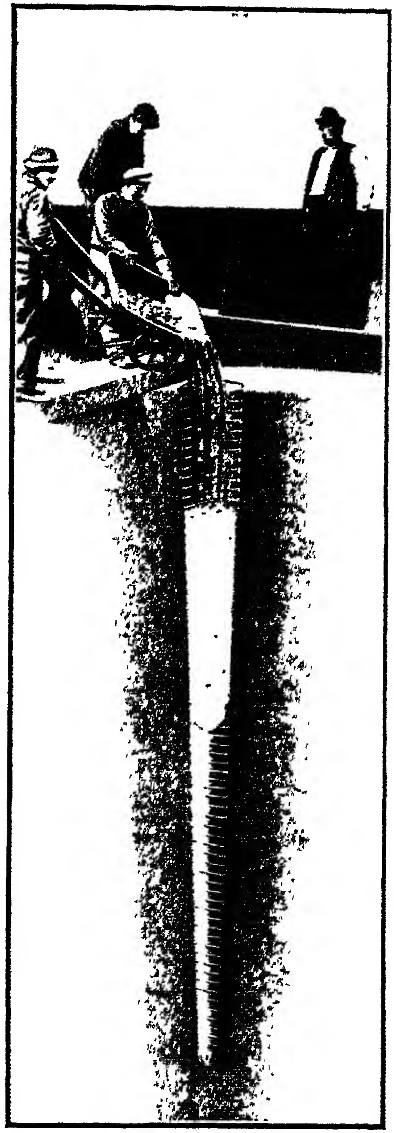


FIG. 117.

Pouring concrete into the Steel Shell or form of a Raymond Concrete Pile. The Shell is left in the ground to protect the setting concrete.

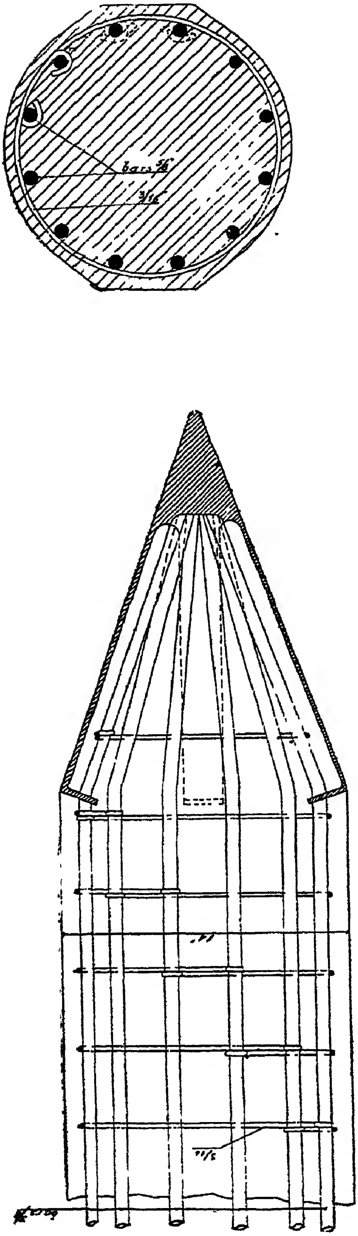


FIG. 118.
Reinforcement and Shoe of a Coignet Pile.

Coignet Piles.—Fig. 118 illustrates a hooped Coignet pile; these have a circular section with two flat sides. This form of pile is claimed by the makers to be the easiest to drive, especially when required to be driven to a great depth; the flat sides are for the purpose of enabling them to be more easily guided during driving.

Square Coignet piles are generally made in a similar manner to circular piles; but four additional bars are sometimes used, one being placed in each corner on the outside of the circular reinforcement, additional hoops are provided for these bars. Fig. 119 shows the completed reinforcement of a Coignet pile, with a cast-iron shoe attached, ready to be placed inside the form. Fig. 120 illustrates the form for one of these piles; the reinforcement is suspended within the form by the bolts connecting the bands.

Another type of reinforced concrete pile is that known as the Chenowith pile; it is made by plastering a woven wire fabric with fine concrete forming a mat, which is then rolled about a mandril into a cylindrical form.

Fig. 119.

Completed Reinforcement of a Coignet Pile.

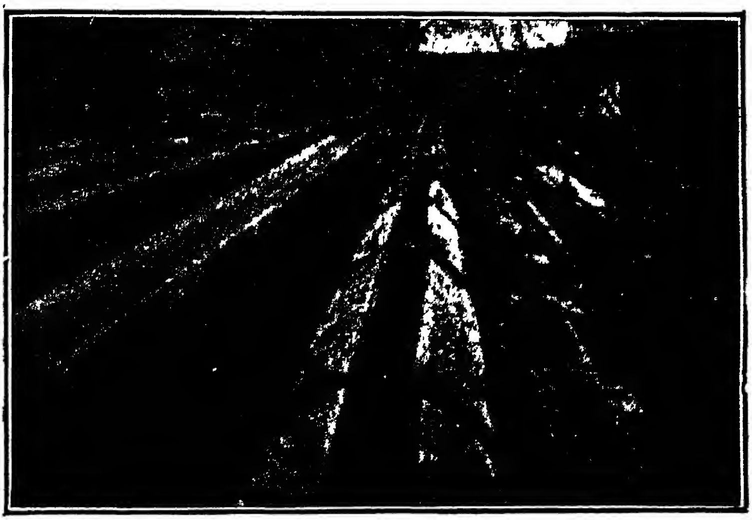


Fig. 120.

Form for a Coignet Circular Pile.

With any method of building reinforced piles in place it is necessary to see that great care is taken in arranging, placing, and fixing the reinforcement so that it will not be displaced by pouring of the concrete, and so that it will not interfere with the necessary working of the concrete to insure absence of voids and maximum density.

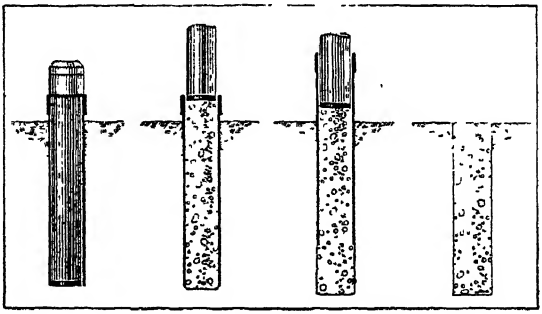


FIG. 121.
Showing various stages of construction of MacArthur Concrete Straight Shaft Pile.

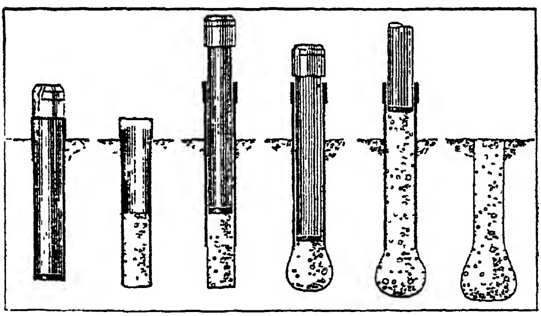


FIG. 122.
Showing various stages of construction of MacArthur Concrete Pedestal Pile.

Other types of concrete piles are the MacArthur Straight Shaft, and the Pedestal Pile. These, however, are not reinforced. The construction of both of these is similar to that of the Raymond Pile ; consisting of a cylindrical shaft of compressed concrete formed by driving a steel casing of 14, 16 or 18 inches in diameter, with a close fitting core into the ground with a steam-

hammer until it indicates the required carrying capacity under the Engineering News Formula. The core is then withdrawn and the casing filled with concrete, having a one-inch slump. As the casing is withdrawn the full weight of the core and hammer rests on the concrete to compact it and force it into the soil.

With the pedestal type, after the core is withdrawn a suitable charge of concrete is placed in the casing, the casing is then pulled up 18 inches to 3 feet, depending on soil conditions, the core is then replaced in the casing, and the charge of concrete hammered out until about 6 inches remains in the casing ; the core is then withdrawn and the casing filled with concrete, having a one-inch slump, to a height above the required elevation of top of the finished pile. The core is again replaced and the casing withdrawn while the core and hammer rests upon the concrete.

Allowable Loads on Piles.—With ordinary cases it is usual to allow for a load on a reinforced concrete pile up to 500 lb. per square inch of section. The building codes of most cities allow from 350 to 500 lb. per square inch for the concrete plus *m* times that amount for the vertical reinforcement ; which gives the same formula for the total load as used in this work for short columns, where $W = c\ [A+(m-1)\ A_c]$. For this load, however, the pile should be driven to refusal, which is not always possible, consequently, some other method of computing the carrying capacity should be used. The most satisfactory way is by applying test loads on piles driven in the particular location, then deciding upon a working load of about one-third to one-half of the test load when on the point of settlement.

Some engineers allow for a load of 30 tons per pile and permits 50 per cent. increase if driven to rock. Others use one of the several well known formulas adopted for timber piles. The most generally used being that known as the Engineering News Formula, which is as follows :—

(1) For piles driven with a drop hammer,

$$W = \frac{2\,w\,h}{p+1}.$$

(2) For piles driven with a single-acting steam hammer

$$W = \frac{2\,w\,h}{p + 0 \cdot 1}.$$

(3) For piles driven with a double-acting steam hammer

$$W = \frac{2\,p\,(w + a\,p)}{p + 0 \cdot 1}.$$

W = total working load for the pile in pounds.
w = weight of the hammer in pounds.
h = height of fall of the hammer in feet.
p = average penetration of the pile in inches under the last few blows.
a = effective area of the piston in square inches.

SHEET PILES.

Sheet piles are generally rectangular in section, with the edges grooved and tongued; they are driven

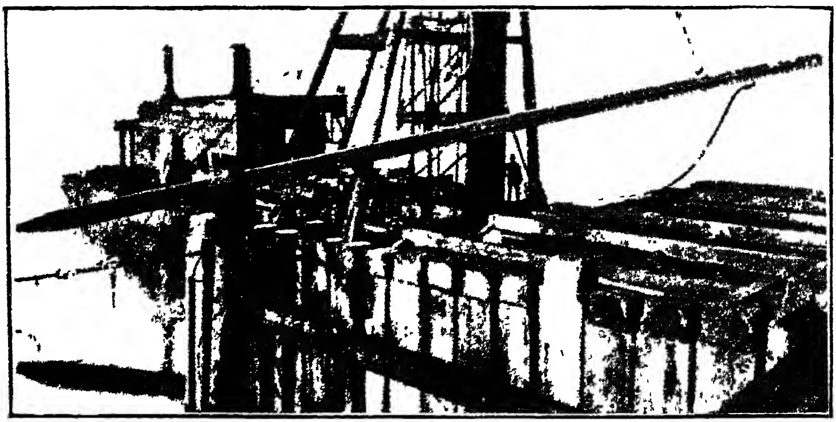


Fig. 123.
Construction of a Bulkhead with interlocking sheet piles 75 feet in length

between guide piles, as in the case of timber or steel sheet piles. They are not very heavily reinforced, as they are usually required to be driven only a few feet into the ground; the greatest pressure they have to resist is generally that due to the retained earth or water, and should therefore be designed in a similar way to retaining walls, for which see Part II.

LIST OF SYMBOLS USED IN THIS WORK.

A	Area of column in square inches.
A_c	Sectional area of steel in compression, in beams and columns.
A	Sectional area of steel in shear.
a	Sectional area, in square inches, of steel in tension.
a'	Sectional area of steel required to take up the excess tension in doubly reinforced beams.
B	Total width of slab, or table of tee beam.
b	Breadth of beam.
c	Maximum compression per square inch in the concrete.
c'	Maximum compression per square inch in the steel.
D	Effective depth of slab.
d	Effective depth of beam. Diameter of column.
d_1	Diameter of column between rods.
E c	Modulus of elasticity of concrete.
E s	Modulus of elasticity of steel.
f, r	Coefficients to reduce the bending moment, or load, on slabs.
g	Distance of the centre of compression from the compression surface in tee beam, and in doubly reinforced beams.
L	Effective length of beam, slab, or column.
l, l', l''	Portion of length of beam, slab, or column.
M	Greatest bending moment.
M′	Bending moment at a given point.
M R	Moment of resistance.
m	Ratio of the moduli of elasticity of steel and concrete $= \frac{E\,s}{E\,c}$
N A	Neutral axis.
n	Distance from compression surface to neutral axis.
p	Ratio of area of steel to area of concrete $= \frac{a}{b\,d}$ or $\frac{c\,n}{2\,t\,d}$

R, R′	Reactions of supports.
S	Greatest shearing stress.
S_1	Shearing stress at a given point.
s	Shearing stress per square inch of section.
t	Maximum tension per square inch in the steel.
t_s	Maximum shear stress per square inch in the steel.
W	Total load.
w	Load per foot of length.
y	Distance from compression surface to steel in compression.
O	Perimeter of bars.
ϕ	Angle of inclination of shear stirrups.

FORMULAS AND THEIR DERIVATIONS.

BEAMS OR SLABS WITH SINGLE REINFORCEMENT.

1. Total compression $= \dfrac{c\,b\,n}{2}$

2. Total tension $= t\,a$

3. Moment of compression $= \dfrac{c\,b\,n\left(d-\frac{n}{3}\right)}{2}$

4. Moment of tension $= t\,a\left(d-\frac{n}{3}\right)$

5. $t = c\,m\,\dfrac{d-n}{n}$

6. $t = \dfrac{M}{a\left(d-\frac{n}{3}\right)}$

7. $t = \dfrac{c\,b\,n}{2\,a}$

8. $m = \dfrac{E\,s}{E\,c}$

9. $p = \dfrac{a}{b\,d}$

10. $$p = \frac{c\,n}{2\,t\,d}$$

11. $$a = p\,b\,d$$

12. $$a = \frac{b\,c\,n}{2\,t}$$

13. $$c = \frac{2\,M}{b\,n\left(d - \frac{n}{3}\right)}$$

14. $$c = \frac{t\,n}{m\,(d-n)}$$

15. When c and t are definite $$n = \frac{m\,c\,d}{t + m\,c}$$

This is derived as follows :

$$c : t :: n : m\,(d-n)$$

therefore

$$\begin{aligned} m\,c\,(d-n) &= t\,n \\ m\,c\,d - m\,c\,n &= t\,n \\ n\,(t + m\,c) &= m\,c\,d \end{aligned}$$

hence

$$\frac{m\,c\,d}{t + m\,c} = n$$

16. When t or c is unknown and the steel is expressed in inches.

$$n = \sqrt{\frac{2\,a\,m\,d}{b} + \left(\frac{a\,m}{b}\right)^2} - \frac{a\,m}{b}$$

Derived as follows :

$c : t :: n : m\,(d-n)$ therefore $c\,m\,(d-n) = t\,n$

Now with the conditions that the total compression and tension are equal, we can equate these to determine n.

Total compression $= \frac{c\,b\,n}{2}$ total tension $= t\,a$ hence $n = \frac{2\,t\,a}{c\,b}$ but here are two unknowns t and c we can, however, replace these with an equivalent for as

$c\ m\ (d-n) = t\ n,\ \frac{c}{t} = \frac{m\ (d-n)}{n}$ by replacing $\frac{c}{t}$ with this equivalent the equation becomes $\frac{2\ a\ m\ (d-n)}{b} = n^2$ an adfected quadratic, which is solved as follows :

$$b\ n^2 = 2\ a\ m\ (d-n)$$

$$b\ n^2 = 2\ a\ m\ d - 2\ a\ m\ n$$

$$b\ n^2 + 2\ a\ m\ n = 2\ a\ m\ d$$

$$n^2 + \frac{2\ a\ m\ n}{b} = \frac{2\ a\ m\ d}{b}$$

$$n^2 + \frac{2\ a\ m\ n}{b} + \left(\frac{a\ m}{b}\right)^2$$

$$= \frac{2\ a\ m\ d}{b} + \left(\frac{a\ m}{b}\right)^2$$

$$\left(n + \frac{a\ m}{b}\right)^2 = \frac{2\ a\ m\ d}{b} + \left(\frac{a\ m}{b}\right)^2$$

$$n + \frac{a\ m}{b} = \sqrt{\frac{2\ a\ m\ d}{b} + \left(\frac{a\ m}{b}\right)^2}$$

Hence

$$n = \sqrt{\frac{2\ a\ m\ d}{b} + \left(\frac{a\ m}{b}\right)^2} - \frac{a\ m}{b}$$

Further simplified,

$$n = \frac{\sqrt{2\ a\ m\ d\ b + (m\ a)^2} - m\ a}{b}$$

17. When t or c is unknown and the steel is expressed in terms of the area of concrete, as p, then

$$n = d\left[\sqrt{2\ p\ m + (p\ m)^2} - p\ m\right]$$

This is derived in a similar manner to No. 16, where it is shown that

$$n^2 = \frac{2\ a\ m\ (d-n)}{b}$$

In this case, however, we have not got a but we know its equivalent is $p\ b\ d$.

Hence by substitution we get :

$$n^2 = \frac{2\,p\,b\,d\,m\,(d-n)}{b}$$

Solving the quadratic we get :

$$n^2 = 2\,p\,d^2\,m - 2\,p\,d\,m\,n$$
$$n^2 + 2\,p\,d\,m\,n = 2\,p\,d^2\,m$$
$$n^2 + 2\,p\,d\,m\,n + (p\,d\,m)^2 = 2\,p\,d^2\,m + (p\,d\,m)^2$$
$$(n + p\,d\,m)^2 = 2\,p\,d^2\,m + (p\,d\,m)^2$$
$$n = \sqrt{2\,p\,d^2\,m + (p\,d\,m)^2} - p\,d\,m$$

Hence $n = d\,[\sqrt{2\,p\,m + (p\,m)^2} - p\,m]$

18. To determine the most economic section of a beam or slab, i.e., such that the concrete and steel can at the same time be stressed to their maximum allowed values.

For stone concrete, when $c = 600$, $t = 16\,000$, $m = 15$, and for brick concrete, when $c = 500$, $t = 16\,000$, $m = 18$.

$M = 0{\cdot}158\ c\ b\ d^2$

$$d = \sqrt{\frac{M}{0{\cdot}158\,c\,b}}$$

Derived as follows :

$$M = \text{the moment of compression} = \frac{c\,b\,n\left(d - \frac{n}{3}\right)}{2}$$

then $$2\,M = b\,c\,n\left(d - \frac{n}{3}\right)$$

and $n = \dfrac{m\,c\,d}{t + m\,c}$ Taking $c = 600$, $t = 16\,000$, $m = 15$

then $$n = \frac{15 \times 600}{16\,000 + 15 \times 600} = 0{\cdot}36\,d$$

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Therefore, substituting the value of n in terms of d we get :—

$$2\ \mathrm{M} = b\ c \times 0{\cdot}36\ d \left(d - \frac{0{\cdot}36\ d}{3} \right)$$

and

$$d\left(d - \frac{0{\cdot}36\ d}{3} \right) = d^2 - 0{\cdot}12\ d^2 = (1 - 0{\cdot}12)\ d^2 = 0{\cdot}88\ d^2$$

Then $\quad 2\ \mathrm{M} = 0{\cdot}36 \times 0{\cdot}88\ c\ b\ d^2$

Hence $\quad \mathrm{M} = 0{\cdot}158\ 4\ c\ b\ d^2$

But c in this equation is 600, and $600 \times 0{\cdot}158\ 4 = 95$ hence $\mathrm{M} = 95\ b\ d^2$ which can be used instead of $0{\cdot}158\ 4\ c\ b\ d^2$. The author has preferred to adopt the former throughout this work, it is, however, optional.

When $c = 600$, $t = 17\ 000$, and $m = 15$

$$\mathrm{M} = 0{\cdot}153\ c\ b\ d^2 \text{ or } 92\ b\ d^2$$

When $c = 650$, $t = 16\ 000$, and $m = 15$

$$\mathrm{M} = 107\ b\ d^2$$

When $c = 500$, $t = 15\ 000$, and $m = 15$

$$\mathrm{M} = 0{\cdot}148\ c\ b\ d^2 \text{ or } 74\ b\ d^2$$

For coke breeze concrete, when

$c = 250$, $t = 16\ 000$, and $m = 30$

$$\mathrm{M} = 0{\cdot}143\ c\ b\ d^2 \text{ or } 36\ b\ d^2$$

The coefficient of $b\ d^2$ for any values of c and t can be determined in the manner shown above or direct by the following formula :—

$$\text{coefficient} = \frac{m\ c^2\ (3\ t + 2\ m\ c)}{6\ (t + m\ c)^2}$$

For single reinforced beams greater or smaller than the economic section, with c fixed but not t.

19. $$n = \frac{3\ d}{2} - \sqrt{\left[\frac{3\ d}{2}\right]^2 - \frac{6\ \mathrm{M}}{c\ b}}$$

20. $$a = \frac{b\,n^2}{2\,m\,(d - n)}$$

19 is derived as follows :—The bending moment equals the moment of compression, therefore

$$M = \frac{c\,b\,n\left(d - \frac{n}{3}\right)}{2}$$

Now M, c, b and d are fixed, we have, therefore, to determine n which we know equals $\frac{m\,c\,d}{t + m\,c}$, but for this case t is unknown ; we can, however, determine n thus :—

$$2\,M = c\,b\,n\left(d - \frac{n}{3}\right).$$

$$6\,M = c\,b\,n\,(3\,d - n).$$

$$3\,d\,c\,b\,n - c\,b\,n^2 = 6\,M.$$

Solving the quadratic we get :—

$$\frac{3\,d\,c\,b\,n}{-c\,b} + n^2 = \frac{6\,M}{-c\,b}.$$

Cancelling c and b, $-3\,d\,n - n^2 = -\frac{6\,M}{c\,b}$.

$$-3\,d\,n - n^2 + -\left(\frac{3\,d}{2}\right)^2 = -\frac{6\,M}{c\,b} + \left[-\frac{3\,d}{2}\right]^2.$$

$$\left[-\frac{3\,d}{2}\right] - n^2 = \left[-\frac{3\,d}{2}\right]^2 - \frac{6\,M}{c\,b}.$$

Hence $$n = \frac{3\,d}{2} - \sqrt{\left[\frac{3\,d}{2}\right]^2 - \frac{6\,M}{c\,b}}.$$

No. 20 is derived as follows :—The total tension equals the total compression ; therefore

$$\frac{c\,b\,n}{2} = t\,a.$$

But t is unknown, it, however, equals

$$\frac{c\ m\ (d-n)}{n}.$$

Then by substitution

$$\frac{c\,b\,n}{2} = \frac{a\,c\,m\,(d-n)}{n}. \qquad c\,b\,n^2 = 2\,a\,c\,m\,(d-n).$$

Hence

$$a = \frac{c\ b\ n^2}{2\ c\ m\ (d-n)} = \frac{b\ n^2}{2\ m\ (d-n)}.$$

BEAMS WITH DOUBLE REINFORCEMENT.

19. Total compression $= \frac{c\,b\,n}{2} + A_c\,c'$

20. Total tension $= t\,a$

21. Moment of compression

$$= \frac{b\,c\,n\left(d-\frac{n}{3}\right)}{2} + A_c\,c'\,(d-y)$$

22. By substituting $c\ m\left(\frac{n-y}{n}\right)$ for c', and $d-g$ for $d-\frac{n}{3}$ and $d-y$ we get,

Moment of compression $= \frac{c\,(d-g)\,[b\,n^2+2\,A_c\,m\ \ (n-y)}{2\,n}$

which can be used instead of 21.
g is determined by 37.

23. Moment of tension $= a\,t\,(d-g)$

24. $t = \frac{M}{a\ (d-g)}$

25. $t = c\,m\,\frac{d-n}{n}$

26. $c' = c\,m\,\frac{n-y}{n}$

27. $a' = \dfrac{\text{excess M}}{t\,(d-y)}$

28. $A_c = \dfrac{\text{excess M}}{c'\,(d-y)}$

29. $A_c = \dfrac{\text{excess M}}{m\,c\,\frac{n-y}{n}(d-y)}$

30. $A_c = a'\dfrac{d-n}{n-y}$

When the compression bars are placed at $\frac{1}{3}\,n$ from the top of the beam.

33. $A_c = \dfrac{3\,(2\,t\,a - c\,b\,n)}{4\,c\,m}$, or without t and a.

34. $A_c = \dfrac{3\left[2\,M - c\,b\,n\left(d - \frac{n}{3}\right)\right]}{4\,c\,m\left(d - \frac{n}{3}\right)}$

33 is derived as follows :—

Equating compression and tension we get,

$$t\,a = \frac{c\,b\,n}{2} + A_{c'}\,m\,c\,\frac{n-y}{n}.$$

$$A_c \text{ being placed at } \frac{n}{3},\quad \frac{n-y}{n} = \frac{2}{3}.$$

Then by substitution, $t\,a = \dfrac{c\,b\,n}{2} + \frac{2}{3}\,A_c\,m\,c.$

$$2\times 3\,t\,a = 3\,c\,b\,n + 2\times 2\,A\,m\,c.$$
$$2\times 3\,t\,a - 3\,c\,b\,n = 4\,A\,m\,c.$$

Hence $$A_c = \frac{3\,(2\,t\,a - c\,b\,n)}{4\,m\,c},$$

34 is derived as follows :—
Equating the bending moment with the moment of compression we get :—

$$M = \left[\frac{b\,c\,n}{2} + \tfrac{2}{3} A_c\, m\, c\right]\left(d - \frac{n}{3}\right).$$

$$3 \times 2M = 3\,b\,c\,n\left(d - \frac{n}{3}\right) + 4\,A_c\,m\,c\left(d - \frac{n}{3}\right).$$

$$3\left[2M - b\,c\,n\left(d - \frac{n}{3}\right)\right] = 4\,A_c\,m\,c\left(d - \frac{n}{3}\right).$$

Hence $$A_c = \frac{3\left[2M - b\,c\,n\left(d - \frac{n}{3}\right)\right]}{4\,m\,c\left(d - \frac{n}{3}\right)}.$$

31. $$c = \frac{2M}{b\,n\left(d - \frac{n}{3}\right) + 2\,A_c\,m\left(\frac{n - y}{n}\right)(d - y)}.$$

No. 31 is derived as follows :

The bending moment equals the moment of compression, which is

$$\frac{b\,c\,n\left(d - \frac{n}{3}\right)}{2} + A_c\, c'\,(d - y) \text{ and } c' = c\,m\,\frac{n - y}{n}$$

then by replacing c' with this equivalent we get

$$2M = c\,b\,n\left(d - \frac{n}{3}\right) + 2\,A_c\,c\,m\,\frac{n - y}{n}(d - y)$$

$$= c\left[b\,n\left(d - \frac{n}{3}\right) + 2\,A_c\,m\,\frac{n - y}{n}(d - y)\right]$$

Hence

$$c = \frac{2M}{b\,n\left(d - \frac{n}{3}\right) + 2\,A_c\,m\left(\frac{n - y}{n}\right)(d - y)}$$

32. By substituting $d - g$ for $d - \frac{n}{3}$ and $d - y$

$$c = \frac{2\,M\,n}{(d - g)\,[b\,n^2 + 2\,A_c\,m\,(n - y)]}$$

33. Moment of the excess tension $= a_c{}'t\,(d - y)$

34. Moment of the compression for the steel $= A_c\,c'\,(d - y)$

$$35.\quad n = \sqrt{\frac{2\,m\,(a\,d + A_c\,y)}{b} + \left(\frac{m\,(A_c + a)}{b}\right)^2} - \frac{m\,(A_c + a)}{b}$$

$$36.\quad n = \frac{\sqrt{2\,m\,b\,(a\,d + A_c\,y) + [m\,(A_c + a)^2]} - m\,(A_c + a)}{b}$$

Nos. 35 and 36 are derived as follows :

Equating tension and compression we get

$$t\,a = \frac{c\,b\,n}{2} + A_c\,c' \text{ therefore } \frac{2\,a\,t}{c} = b\,n + \frac{2\,A_c\,c'}{c}$$

We have here four unknowns, t, c, n and c'

We can, however, replace t, c and c' with equivalents as

$$\frac{t}{c} = \frac{m\,(d - n)}{n} \text{ and } c' = \frac{c\,m\,(n - y)}{n}$$

or

$$\frac{c'}{c} = \frac{m\,(n - y)}{n}$$

The equation then becomes

$$\frac{2\,a\,m\,(d - n)}{n} = b\,n + \frac{2\,A_c\,m\,(n - y)}{n}$$

an adfected quadratic which is solved as follows :

$$\begin{aligned} b\,n^2 + 2\,A_c\,m\,(n - y) &= 2\,a\,m\,(d - n) \\ b\,n^2 + 2\,A_c\,m\,n - 2\,A_c\,m\,y &= 2\,a\,m\,d - 2\,a\,m\,n \\ b\,n^2 + 2\,n\,m\,(A_c + a) &= 2\,m\,(a\,d + A_c y) \end{aligned}$$

$$n^2 + \frac{2\,n\,m\,(A_c+a)}{b} + \left[\frac{m\,(A_c+a)}{b}\right]^2$$

$$= \frac{2\,m\,(a\,d+A_c\,y)}{b} + \left[\frac{m\,(A_c+a)}{b}\right]^2$$

$$\left[n + \frac{m\,(A_c + a)}{b}\right]^2 = \frac{2\,m\,(a\,d + A_c\,y)}{b} + \left[\frac{m\,(A_c + a)}{b}\right]^2$$

Hence

$$n = \sqrt{\frac{2\,m\,(a\,d + A_c\,y)}{b} + \left[\frac{m\,(A_c + a)}{b}\right]^2} - \frac{m\,(A_c + a)}{b}$$

Further simplified,

$$n = \frac{\sqrt{2\,m\,b\,(a\,d+A_c\,y)+[m\,(A_c+a)]^2} - m\,(A_c+a)}{b}$$

37. $$g = \frac{b\,n^3 + 6\,A_c\,m\,y\,(n-y)}{3\,[b\,n^2 + 2\,A_c\,m\,(n-y)]}$$

Economic section for equal or unequal reinforcement. When $A_c = a$, $c = 600$, and $t = 16\,000$.

38. $M = 0{\cdot}2\,534\,c\,b\,d^2$, or $152\,b\,d^2$.

39. $$p = \frac{n}{d}\,\frac{3\,c}{6\,t - 4\,m\,c} = 0{\cdot}0\,108.$$

40. $a = p\,b\,d$ (p being for the tensile reinforcement only, the total proportion $= 2\,p$). Or

41. $$a = \frac{3\,c\,b\,n}{6\,t - 4\,m\,c}$$

~~38~~ 41 is derived as follows :—Equating compression and tension we get :

$$\frac{c\,b\,n}{2} + \tfrac{2}{3}\,A_c\,m\,c = t\,a.$$

But $$A_c = a,$$

therefore
$$3\,c\,b\,n + 4\,a\,m\,c = 6\,t\,a.$$
$$3\,c\,b\,n = 6\,t\,a - 4\,a\,m\,c.$$

Hence $$a = \frac{3\,c\,b\,n}{6\,t - 4\,m\,c}.$$

39 is derived as follows :— $a = p\,b\,d$, therefore $p = \dfrac{a}{b\,d}$.

Then p equals equation 41, divided by $b\,d$.

Hence $$p = \frac{b\,d\,(6\,t - 4\,m\,c)}{3\,c\,b\,n} = \frac{n}{d}\,\frac{3\,c}{6\,t - 4\,m\,c}.$$

38 is derived as follows :—

$$M = t\,a\left(d - \frac{n}{3}\right) = t\,a\left(\frac{3\,d - n}{3}\right), \text{ but } a = \frac{3\,b\,c\,n}{6\,t - 4\,m\,c}.$$

Then by substitution,

$$M = \frac{3\,c\,b\,n\,t\left(\dfrac{3\,d - n}{3}\right)}{6\,t - 4\,m\,c} = \frac{c\,b\,n\,t\,(3\,d - n)}{6\,t - 4\,m\,c}$$

$$= \frac{t\,c\,b\left(\dfrac{n\,d}{d}\right)\left(3\,d - \dfrac{n\,d}{d}\right)}{6\,t - 4\,m\,c}$$

Hence $$M = \frac{c\,b\,d^2\,t\,\dfrac{n}{d}\left(3 - \dfrac{n}{d}\right)}{6\,t - 4\,m\,c},$$

Or $$\frac{M}{c\,b\,d^2} = \frac{n}{d}\,\frac{t\left(3 - \dfrac{n}{d}\right)}{6\,t - 4\,m\,c}.$$

Now $$\frac{n}{d} = \frac{m\,c}{t + m\,c}.$$

Then for $c = 600$, $t = 16\,000$. $\dfrac{n}{d} = 0{\cdot}36$.

Substituting these values we get :—

$$M = c\,b\,d^2 \frac{16\,000 \times 0{\cdot}36\,(3 \;-\; 0{\cdot}36)}{6 \times 16\,000 - 4 \times 15 \times 600} = 0{\cdot}2534\,c\,b\,d^2.$$

Or $152\,b\,d^2$.

If we have a value for b, then

$$d = \sqrt{\frac{M}{152\,b}}.$$

If we have no value for b, we can make it some ratio of d, such as $0{\cdot}6\,d$, then

$$d = \sqrt[3]{\frac{M}{152 \times 0{\cdot}6}}$$

and $b = 0{\cdot}6\,d$. For $c = 600$, and $t = 17\,000$, $M = 144\,b\,d$, and $p = 0{\cdot}00954$.

For any other values of c and t the value of $\frac{M}{c\,b\,d^2}$ can be determined as shown for equation 38, p by 39, and a by 41.

If A_c is required to be some other ratio of a, say $0{\cdot}6\,a$, equation 41 becomes :—

$$a = \frac{3\,c\,b\,n}{6\,t - 0{\cdot}6 \times 4\,m\,c} \qquad (42),$$

and $A_c = 0{\cdot}6\,a$.

Equation 39 becomes :—

$$p = \frac{n}{d}\,\frac{3\,c}{6\,t - 0{\cdot}6 \times 4\,m\,c} \qquad (43).$$

If the section of concrete is fixed, and is more than required to give the economic section, the tensile reinforcement can be determined by equation,

$$a = \frac{M}{t\left(d - \frac{n}{3}\right)},$$

as for beams with single reinforcement.

SLABS.

Factors for Reduction of the Bending Moment, or Load.

38. $f \quad = \quad \dfrac{0 \cdot 05 \text{ B} + 0 \cdot 45 \text{ L}}{\text{B}} \qquad \text{or } 1 - r$

39 $r \quad = \quad \dfrac{0 \cdot 95 \text{ B} - 0 \cdot 45 \text{ L}}{\text{B}} \qquad \text{or } 1 - f$

TEE BEAMS.

When the neutral axis is above the bottom of the slab the formulae for rectangular beams will apply.

When the Neutral Axis is below the Slab.

40. Total tension $= t\, a$

41. Total compression

$$= \text{B D} \left(c + c \frac{n - \text{D}}{n}\right) + \frac{b\, c\, (n - \text{D})^2}{n}$$

42. Moment of compression

$$= \frac{(d - g)\, c\, [\text{B D } (2\, n - \text{D}) + b\, (n - \text{D})^2]}{2\, n}$$

g is determined by 47 or 48.

43. Moment of tension $= t\, a\, (d - g)$

44. $$t = \frac{\text{M}}{a\, (d - g)}$$

45. $$c = \frac{2 \text{ M } n}{(d - g)\, [\text{B D } (2\, n - \text{D}) + b\, (n - \text{D})^2]}$$

46. $$c = \frac{t\, n}{m\, (d - n)}$$

47. Neglecting the small compression in the leg,

$$g = \frac{3 \text{ D } n - 2 \text{ D}^2}{6\, n - 3 \text{ D}}$$

48. Considering the compression in the leg,

$$g = \frac{B\,D^2\,(3\,n - 2\,D) + b\,(2\,D + n)\,(n - D)^2}{3\,[B\,D\,(2\,n - D) + b\,(n - D)^2]}$$

49. $$n = \sqrt{\frac{2\,m\,a\,d + D^2\,(B - b)}{b} + \left[\frac{m\,a + D\,(B - b)}{b}\right]} - \frac{m\,a + D\,(B - b)}{b}$$

SHEARING FORMULAE.

50. $$s = \frac{S}{b\left(d - \frac{n}{3}\right)}$$

51. Adhesive stress per square inch $= \dfrac{S}{o\left(d - \frac{n}{3}\right)}$

52. For vertical stirrups, $A_s = \dfrac{S}{t_s}$

53. For inclined stirrups, $A_s = \dfrac{S}{\sqrt{2}\,t}$

COLUMNS.

54. For short columns, $W = c\,(A + 14\,A_c)$

55. For short columns, $W = A\,(c + 14\,p\,c)$

or $A\,c\,(1 + 14\,p)$

56. For long columns, $W = \dfrac{c\,(A + 14\,A_c)}{1 + \dfrac{k\,l^2}{u\,d^2}}$

57. $$A_c = \frac{W\left(1 + \frac{k\,l^2}{u\,d^2}\right) - A\,c}{14\,c}$$

ECCENTRICALLY LOADED COLUMNS.

58. Max $c \quad = \dfrac{W}{A^1} + \dfrac{W\,y}{Z}$

59. Min. $c \quad = \dfrac{W}{A^1} - \dfrac{W\,y}{Z}$

60. W $\quad = \dfrac{\text{Max. } c\ A^1\ Z}{Z + A^1\ y}$

61. For a rectangular column, $Z = \dfrac{b\ d^3 + 42\ A_c\ d_1^2}{6\ d}$

62. For a circular column, with the bars arranged to form a rectangle, $Z = \dfrac{d^4 + 56\ A_c\ d_1^2}{8\ d}$

63. For a circular column, with the bars arranged in a circle,

$$Z = \frac{d^4 + 28\ A_c\ d_1^2}{8\ d}$$

AREAS OF SQUARE AND ROUND BARS AND CIRCUMFERENCE OF ROUND BARS.

Thickness or Diameter in Inches	Area of ■ Bar in Sq. Inches	Area of ● Bar in Sq. Inches	Circumference of ○ Bar in Inches	Thickness or Diameter in Inches	Area of ■ Bar in Sq. Inches	Area of ● Bar in Sq. Inches	Circumference of ○ Bar in Inches
				$\frac{3}{4}$	0·562 5	0·441 8	2·356 2
				$\frac{13}{16}$	0·660 2	0·518 5	2·552 6
$\frac{1}{16}$	0·003 9	0·003 1	0·196 4	$\frac{7}{8}$	0·765 6	0·601 3	2·748 9
$\frac{3}{32}$	0·008 8	0·006 9	0·294 5	$\frac{15}{16}$	0·878 9	0·690 3	2·945 3
$\frac{1}{8}$	0·015 6	0·012 3	0·392 7	1	1·000 0	0·785 4	3·141 6
$\frac{5}{32}$	0·024 4	0·019 2	0·490 9	$1\frac{1}{16}$	1·128 9	0·886 6	3·338 0
$\frac{3}{16}$	0.035 2	0·027 6	0·589 1	$1\frac{1}{8}$	1·265 6	0·994 0	3·534 3
$\frac{7}{32}$	0·047 9	0·037 6	0·687 2	$1\frac{3}{16}$	1·410 2	1·107 5	3·730 6
$\frac{1}{4}$	0·062 5	0·049 1	0·785 4	$1\frac{1}{4}$	1·562 5	1·227 2	3·927 0
$\frac{9}{32}$	0·079 1	0·062 1	0·883 6	$1\frac{5}{16}$	1·722 7	1·353 0	4·123 4
$\frac{5}{16}$	0·097 7	0·076 7	0·981 8	$1\frac{3}{8}$	1·890 6	1·484 9	4·319 7
$\frac{11}{32}$	0·118 2	0·092 8	1·079 9	$1\frac{7}{16}$	2·066 4	1·623 0	4·516 1
$\frac{3}{8}$	0·140 6	0·110 4	1·178 1	$1\frac{1}{2}$	2·250 0	1·767 1	4·712 4
$\frac{13}{32}$	0·165 0	0·129 6	1·276 3	$1\frac{9}{16}$	2·441 4	1·917 5	4·908 8
$\frac{7}{16}$	0·191 4	0·150 3	1·374 5	$1\frac{5}{8}$	2·640 6	2·073 9	5·105 1
$\frac{15}{32}$	0·219 7	0·172 6	1·472 6	$1\frac{11}{16}$	2·847 7	2·236 5	5·301 5
$\frac{1}{2}$	0·250 0	0·196 3	1·570 8	$1\frac{3}{4}$	3·062 5	2·405 3	5·497 8
$\frac{17}{32}$	0·282 2	0·221 7	1·669 0	$1\frac{13}{16}$	3·285 2	2·580 2	5·694 2
$\frac{9}{16}$	0·316 4	0·248 5	1·767 2	$1\frac{7}{8}$	3·515 6	2·761 2	5·890 5
$\frac{19}{32}$	0·352 5	0·276 9	1·865 3	$1\frac{15}{16}$	3·753 9	2·948 3	6·086 9
$\frac{5}{8}$	0·390 6	0·306 8	1·963 5	2	4·000 0	3·141 6	6·283 2
$\frac{21}{32}$	0·430 7	0·338 2	2·061 7				
$\frac{11}{16}$	0·472 7	0·371 2	2·159 9				
$\frac{23}{32}$	0·516 6	0·405 7	2·258 0				

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